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Mix $\star$-autonomous quantales and the continuous weak order

The set of permutations on a finite set can be given the lattice structure known as the weak Bruhat order. This lattice structure is generalized to the set of words on a fixed alphabet $\Sigma = \{ x, y, z, \ldots \}$, where each letter has a fixed number of occurrences. These lattices are known as multinomial lattices and, when $\mathrm{card}(\Sigma) = 2$, as lattices of lattice paths. By interpreting the letters $x, y, z, \ldots$ as axes, these words can be interpreted as discrete increasing paths on a grid of a d-dimensional cube, with $d = \mathrm{card}(\Sigma)$.

In this talk I'll explain how to extend this order to images of continuous monotone functions from the unit interval to a d-dimensional cube. The lattice so obtained is denoted $\mathsf{L}(\mathbb{I}_d)$. The key tool used to realize this construction is the quantale $\mathsf{L}_\vee(\mathbb{I})$ of join-continuous functions from the unit interval to itself; the construction relies on a few algebraic properties of this quantale: it is cyclic $\star$-autonomous and it satisfies the mix rule.

We begin developing a structural theory of the lattices $\mathsf{L}(\mathbb{I}_d)$: they are self-dual, they are generated under infinite joins from their join-irreducible elements, they have no completely irreducible elements. The colimit of all the d-dimensional multinomial lattices embeds into $\mathsf{L}(\mathbb{I}_d)$. When $d = 2$, $\mathsf{L}(\mathbb{I}_d) = \mathsf{L}_\vee(\mathbb{I})$ is the Dedekind-MacNeille completion of this colimit. When $d \geq 3$, every element of $\mathsf{L}(\mathbb{I}_d)$ a join of meets of elements from this colimit.

References

[1] M. J. Gouveia, L. Santocanale, *Mix $\star$–autonomous quantales and the continuous weak order*, 2018, to appear in the proceedings of the conference RAMICS 2018. Preprint available on Hal: https://hal.archives-ouvertes.fr/hal-01838560

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