

*Partial frames and filter spaces*

In general topology, filters have proved a popular tool for the construction of completions. In pointfree topology, downsets are frequently employed for the same purpose. Indeed, any downset frame can be viewed as the topology of the space of filters on that frame. In this paper we consider such ideas in the setting of partial frames; these are meet-semilattices where, in contrast with frames, not all subsets need have joins; a so-called selection function specifies those subsets that must have joins.

We have made use of the lattice of certain kinds of downsets in several situations; to understand the spatial aspect of these downsets, we are led naturally to consider the notion of partial spaces as a generalization of topological spaces. We show that a suitable collection of downsets is isomorphic to the collection of opens of an appropriate filter space, and investigate issues of functoriality.

Next we consider questions of spatiality for the congruence frame of a partial frame. Quotients for frames can be provided using nuclei or congruences but in the setting of partial frames nuclei do not suffice, whereas congruences do.

Whether the congruence lattice of a partial frame is itself a frame or not depends on the axioms assumed for the selection function under consideration: if at least all finite subsets are selected, the congruence lattice is indeed a frame. For such, we characterize the spatiality of the frame of all congruences on a partial frame in terms of its quotients.