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Metagories

Metagories generalize metrically enriched categories. They still have objects and hom-sets that are Lawvere [1] metric spaces, as well as designated identity morphisms, but they are lacking a composition law. Thinking of morphisms $f\colon x \to y$ and $g\colon y \to z$ as paths between vertices x, y, z, rather than being able to compose them, for every path $h\colon x \to z$ we are only given a *content* value for the "directed triangle" spanned by f, g, h. The content function must obey two unit laws and two *tetrahedral inequalities* which mimic associativity.

These inequalities appear in Gähler's *2-metric spaces* [2] which come with a function that is to be thought of as measuring the area of the triangle with vertices x, y, z; in a metagory, we think of the straight edges as being traded for specified curvy directed paths. This idea is due to Aliouche and Simpson [3] whose notion of *approximate categorical structure* gives a marginally tighter concept than that of a metagory. But we achieve substantially greater generality by allowing the content function of a metagory to assume values in an arbitrary commutative quantale $\mathcal{V}$.

Extending part of the work of [3], we show that every so-called transitive $\mathcal{V}$-metagory may be isometrically mapped into a genuine $\mathcal{V}$-metrically enriched category, via a metagorical Yoneda embedding. To this end we first develop the metagorical counterpart of the notion of natural transformation, which is missing from [3].

References

[1] F.W. Lawvere, Metric spaces, generalized logic, and closed categories, *Rend. Sem. Mat. Fis. Milano* 43 (1973) 135-166. Author-commented version available in: *Reprints in Theory and Applications of Categories* 1 (2002) 1-37.

[2] W. Gähler, 2-metrische Räume und ihre topologische Struktur. *Mathematische Nachrichten* 26 (1963) 115-148.

[3] A. Aliouche and C. Simpson, Approximate categorical structures, *Theory and Applications of Categories.* 32(49) (2017) 1522-1562.

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