

Topological properties of non-Archimedean approach spaces and Shanin's compactification

Inspired by the description of approach spaces as lax algebras for the ultrafilter monad β , laxly extended to P_+ [2], we give a lax-algebraic characterization of the category of non-Archimedean approach spaces by interchanging the quantale P_+ by P_V .

We translate this abstract characterization in terms of limit operators, distances and towers of topologies, as introduced in [1]. We also add a new characterization using the gauge.

Secondly we locate the category (β, P_V) -Cat relative to more familiar categories Top , qMet^u and App via reflective and/or coreflective embeddings.

Furthermore we investigate topological properties p in (β, P_V) -Cat, following the relational calculus developed in [4]. For p a topological property we investigate the link between (β, P_V) - p and the property p for the level topologies \mathcal{T}_ε of the non-Archimedean tower $(\mathcal{T}_\varepsilon)_{\varepsilon \in \mathbb{R}^+}$.

To conclude we consider the case of compact Hausdorff non-Archimedean approach spaces in detail and we describe a method for constructing a compactification based on Shanin's method.

References

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