Chapter 1
Modeling bimaterial 3D printing using galvanometer mirror scanners

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Abstract Three-dimensional printing is a process for building new parts with a specified shape. Despite its increasing popularity, printers capable of working with more than one material are yet unavailable. In this work we model the design and the operation of an apparatus for printing with two materials, namely printing a component which includes a previously constructed inner structure. The structure that supports the second material brings difficulties, resulting from the possible “shaded” areas on the printing surface. The problem is addressed assuming the instalation of galvanometer mirror scanners as additional light sources on the walls of the printer, and it is modeled in two steps: finding the least number of emitters to use, so that the whole part can be constructed, as well as their position; and assigning them with each cell of the part to be reached. The first step is formulated as a set covering problem. The second is formulated as a linear integer problem and aiming at optimizing two objectives: the number of emitters activated per layer and the quality of the printed part. Methods for solving the problems are described and tested.

1.1 Introduction

Three-dimensional printing, or 3D printing, is an additive process for rapid free form manufacturing, where the final object (known as part) is created by the addition of successive thin layers of material. Each layer corresponds to a cross-section of the part to be constructed, and the printer draws each layer as if it was a 2D printing [2, 6]. Printings are made of a single material, which can vary from resin to ceramics or metal (among others). One of the technologies used for 3D printing is stereolithography (SL). In this case, each layer is added using liquid resin exposed to a laser light, usually fixed at the top of the printer and able to reach the printing platform. Only the zone of the resin that is reached by the laser beam is cured. Then the platform that supports the model moves to get ready for printing the next layer. This type of 3D printing is fairly standard nowadays, and has become quite popular due to its ability to produce new parts quickly and at a low cost.

The present work focuses on a process analogous to SL, but with the goal of printing an object in which the resin covers a previously constructed 3D grid structure of a different material, like metal. This type of components has application to custom orthotics, intelligent components, complex or fragile parts where over-injection or others options are not feasible or not economically sustainable. In this case the metal structure may block the laser light, thus preventing the cure of shaded areas. This work addresses the question of placing additional galvanometer mirror scanners on the walls of the printer to overcome this issue. Their positions depend on the part to print and are fixed from the beginning of the printing process. However, the laser beam reflected by each galvanometer scanner can be oriented with the goal of reaching the shaded areas. Hereafter galvanometer scanners and laser are sometimes simply referred to as emitters.

As explained before, the part is divided into layers, each one evenly partitioned in squares, called voxels. Assuming that both the voxels to cure and the possible locations for the emitters are known, the problem is modeled in two parts:

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- Emitters location problem (ELP): The goal of which is to find the emitters' position that minimizes the number of emitters required to complete the printing.
- Emitters assignment problem (EAP): Using the solution of the ELP, it is then necessary to determine the voxels of each layer that each emitter should reach.

The rest of the text is organized as follows. In Sections 1.2 and 1.3 integer linear optimization models are presented for these two problems. The formulations are empirically tested for a case study in Section 1.4. Finally, concluding remarks are discussed.

1.2 Emitters location problem

The goal of the ELP is to find the minimum number of emitters that allows to print a given part, as well as their location. To do this, let us consider that there are m voxels that the laser light needs to cure and n possible positions for the laser emitters. The emitters coverage matrix is defined as $A = [a_{ij}]_{i=1,\dots,m;j=1,\dots,n}$, such that

$$a_{ij} = \begin{cases} 1 \text{ if the emitter at position } j \text{ can reach the voxel } i \\ 0 \text{ otherwise} \end{cases}, \quad i = 1,\dots,m, \; j = 1,\dots,n.$$

The matrix A can be calculated by geometric arguments, as shown in [1]. Let also x_j be binary decision variables, such that

$$x_j = \begin{cases} 1 \text{ if the emitter at position } j \text{ is installed} \\ 0 \text{ otherwise} \end{cases}, \; j = 1,\dots,n.$$

The objective function of the ELP is the total number of emitters to use, which is to be minimized. This is given by

$$\sum_{j=1}^{n} x_j.$$

We say that the emitter j covers the voxel i, or that i is covered by j, if it is able to reach it by means of a laser beam, for any $j = 1,\dots,n$, $i = 1,\dots,m$. At least one emitter needs to cover each voxel, in order to cure the material. Therefore, a solution for the ELP is feasible if any voxel i, can be reached by at least one emitter, that is, if

$$\sum_{j=1}^{n} a_{ij} x_j \geq 1, \;\; i = 1,\dots,m.$$

Thus, the ELP can be formulated as the set covering problem below,

$$\begin{array}{ll} \text{minimize} & \sum_{j=1}^{n} x_j \\ \text{subject to} & Ax \geq 1 \\ & x \in \{0,1\}^n \end{array} \tag{1.1}$$

The optimal value of problem (1.1) is the number of emitters required to ensure the complete printing of the part. Its optimal solution provides the positions where the emitters should be installed, which corresponds to the indices j such that $x_j = 1$, $j = 1,\dots,n$. The set covering problem is a classical combinatorial optimization problem which has been shown to be NP-complete [3, 7], therefore, exact methods may be limited to solve it as the size of the problem grows.

1.3 Emitters assignment problem

Assume now that n_2 emitters have been installed in the positions determined by the ELP. The goal of the EAP is then to select the emitter to assign to each voxel. Also consider that p layers of the part need to be printed, each one with m_k voxels to cure, $k = 1,\dots,p$.

Let k be a layer to print, $k = 1,\dots,p$, x_j be variables similar to those used in the ELP, and y_{ij} be binary variables defined by

$$y_{ij} = \begin{cases} 1 \text{ if the emitter } j \text{ is activated to cure voxel } i \\ 0 \text{ otherwise} \end{cases}, \; i = 1,\dots,m_k, \; j = 1,\dots,n_2.$$

Two aspects are taken into account for defining the objective functions, the operability of the system and the quality of the printing. The first is expressed by the number of emitters used on each layer of the part, and the second as the distortion of the laser light when it reaches the layer, denoted by z_1 and z_2, respectively. With this respect it should be noted that the light beam has the shape of a circle at its origin, but the circle is distorted as an ellipse when reaching the layer, every time its incidence angle is not exactly 90º. Therefore, similarly to the ELP, the first criteria, to minimize, can be expressed by

$$z_1(x,y) = \sum_{j=1}^{n_2} x_j.$$

The second depends on the emitter that reaches each voxel, y_{ij}, and the beam's angle of incidence, $\theta_{ij} \in [0, \frac{\pi}{2}]$, $i = 1,\dots,m_k$, $j = 1,\dots,n_2$, which can be calculated by a procedure similar to the emitters coverage matrix [1] – see Figure 1.3a. Thus, minimizing the distortion of the laser light corresponds to maximizing the function

$$z_2(x,y) = \sum_{i=1}^{m_k} \sum_{j=1}^{n_2} \theta_{ij} y_{ij}.$$

The choice of the emitter used to reach a particular voxel must take two aspects into account: the uniqueness of this solution and its viability. Assignment constraints can be used to ensure the first one

$$\sum_{j=1}^{n_2} y_{ij} = 1, \; i = 1,\dots,m_k, \tag{1.2}$$

whereas the second depends on the constraints

$$\sum_{j=1}^{n_2} a_{ij} y_{ij} = 1, \; i = 1,\dots,m_k, \tag{1.3}$$

where $A = [a_{ij}]_{i=1,\dots,m_k; j=1,\dots,n_2}$ is the submatrix of the emitters coverage matrix, restricted to the set of voxels to cure at layer k and the emitters installed in the printer. The other aspect to consider is the emitters that are activated to print the k-th layer. For the emitters used in each layer, the covering conditions introduced in Section 1.2 can be used,

$$\sum_{j=1}^{n_2} a_{ij} x_j \geq 1, \; i = 1,\dots,m_k. \tag{1.4}$$

The variables y_{ij} and x_j are related, because when the emitter j is activated to reach a voxel i, it is activated for the entire layer. This corresponds to imposing the constraints

$$y_{ij} \leq x_j, \; i = 1,\dots,m_k, \; j = 1,\dots,n_2,$$

and, by aggregating these conditions, we can derive the equivalent constraints

$$\sum_{i=1}^{m_k} y_{ij} \leq m_k x_j, \; j = 1,\dots,n_2. \tag{1.5}$$

Finally, it should be noted that when (1.3) and (1.5) hold, the constraints (1.4) are satisfied as well. Combining all the information, we obtain the following biobjective linear integer problem,

$$
\begin{aligned}
\text{minimize} \quad & z_1(x,y) \\
\text{maximize} \quad & z_2(x,y) \\
\text{subject to} \quad & (1.2),(1.3),(1.5) \\
& y_{ij} \in \{0,1\},\ i=1,\ldots,m_k,\ j=1,\ldots,n_2 && (1.6a) \\
& x_j \in \{0,1\},\ j=1,\ldots,n_2. && (1.6b)
\end{aligned}
$$

In general, the objective functions z_1 and z_2 are conflicting, which means that there is no feasible solution that optimizes both simultaneously. Solving the problem considering only z_1 or z_2 provides an idea of how much the two objective functions may range, but as an alternative to the usual concept of optimality in single-objective problems, in these cases we usually seek for efficient solutions. A solution is said to be efficient if there is no other which is strictly better than the first with respect to the two objective functions simultaneously. The approaches for finding the efficient solutions of biobjective integer problems can be classified into a priori, interactive or a posteriori, depending on whether how one efficient solution is chosen among the all set. In the first case the decision maker (DM) knows how the relative importance of the two objective functions, which can be aggregated accordingly before one solution is found. In the second case, partial efficient solutions are shown to the DM, who then guides the search for an acceptable solution. The last case consists of computing all the efficient solutions and then let the DM express the preferences with respect to that whole set. A single solution must be chosen before printing a given part, however it is not clear in advance how the two objective functions are related, thus an a posteriori approach is more indicated for the EAP. Several methods have been proposed to find the efficient solutions of biobjective integer problems like the EAP. For instance, two-phase methods or the ε-constrained method, among others [4, 5]. This topic will be the subject of future research.

1.4 Computational experiments

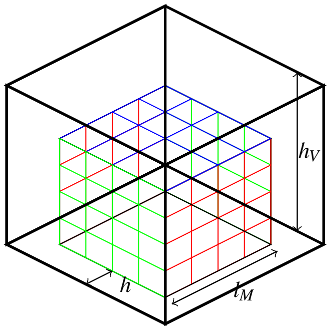
The formulations presented above were tested for a case study consisting of constructing the cube with 5 faces shown in Figure 1.1a. The thickness of the metal grid is considered equal to the thickness of the resin layers, that is, 1. This is also the value used as the width of the voxels. The parameters for printing the cube are:

- The length of each segment of the metal grid, l_M.
- The thickness of the resin added on each side of the metal grid, $l_P = 1$.
- The number of divisions of the metal grid, which is assumed to be uniform, n_M.
- The distance between the cube to print and the side walls of the printer, where emitters can be installed, h, which depends on the size of the part, but ensuring that the printing platform is of size 1250×1250 units.
- The height of the printing area, fixed to $h_V = 1250$ units.

The used length unit corresponds to 0.2 millimeters, the length of the side of the voxels. Each layer contains $n_V \times n_V$ voxels. The remaining characteristics of the problems solved are summarized in Figure 1.1b. The linear problems were solved using CPLEX 12.7, whereas MATLAB R2016b was used for the remaining calculations. The presented run times are mean values obtained for 30 repetitions on an Intel® i7-6700 Quadcore of 3.4 GHz, with 8Mb of cache and 16 Gb of RAM.

For the ELP it was assumed that a laser is already fixed at the center of the top of the printer. Additionally, 80 possible locations are considered for other emitters on the side walls. The solutions of problem (1.1) given by CPLEX are presented in Table 1.1. According to the results, between 2 and 4 emitters besides the top one are required for completing the printing. Although most problems were solved in less than 3 minutes, one of them required almost half an hour, which reflects the hardness of the problem.

Using the solutions in Table 1.1, the EAP was considered when optimizing one objective function at a time. The approach that optimizes z_i is represented below by A_i, $i = 1,2$. Figure 1.2 shows the mean results for the number of non fixed emitters required for printing each layer, μ_1, the mean value of θ, μ_2, and the run times regarding printing the whole part for each method. In terms of solutions the approach A_1 always finds a way to print the part using 2 or 3 emitters per layer besides the top one, while this only happens with A_2 when a broader grid is considered. For the remaining cases applying A_2 implies using 3 or 4 emitters. The average angles of incidence of the beam over the voxels are between 60^{o} and 80^{o} when using approach A_5 and between 45^{o} and 80^{o} when using A_1. The results are worse, i.e., the angle of incidence is smaller, when the grid are denser. The approach A_1 is more sensitive to this change than A_5. As explained next, small angles of incidence may lead to a distorted final part. In general in average the two approaches run fast, a few seconds, and in approximately the same CPU time. However, the tests T3 and T8 were harder to solve using the approach A_1 than using A_2, around 30 and 5 minutes, respectively.



(a) Printing area and object to print

Test	n_V	n_M	h
T1	200	5	525
T2	200	10	525
T3	200	20	525
T4	300	5	475
T5	300	10	475
T6	300	20	475
T7	500	5	375
T8	500	10	375
T9	500	20	375

(b) Test parameters

Fig. 1.1: Case study

Table 1.1: ELP solutions and run times

Test	Emitters' positions	Time (s)
T1	(1, 1000, 250) and (1250,1,250)	5.03
T2	(1, 1000, 1000), (1, 251,250) and (1250, 1250, 250)	19.13
T3	(1250, 1, 1000), (1, 1, 250), (1, 1250, 250) and (1250, 1250, 250)	140.03
T4	(1, 1, 750) and (1250, 1250, 250)	9.89
T5	(1, 750, 1000), (1, 251, 500) and (1250, 1000, 500)	34.98
T6	(1, 1, 500), (1250, 1250, 500), (1, 750, 250) and (1250, 501, 250)	1323.36
T7	(1, 1, 750) and (1250, 1250, 500)	27.52
T8	(1, 1, 500), (1250, 1250, 500) and (1250, 1, 250)	40.70
T9	(1, 1, 500), (1250, 1250, 500) and (1250, 1, 250)	144.05

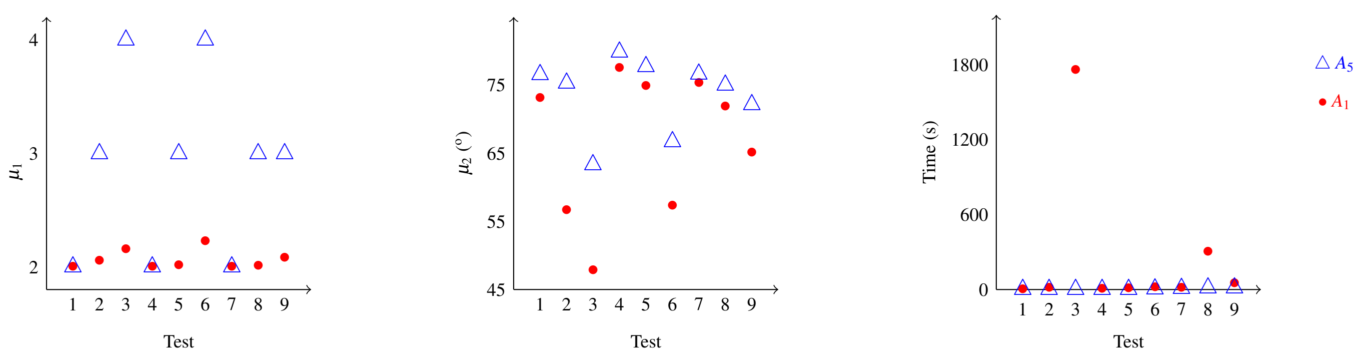


Fig. 1.2: EAP solutions and run times

A measure of the quality of the produced part should also be taken into account. As mentioned earlier, in general, the laser beam reaches the printing surface as an ellipse because of angle θ. We have considered that the centers of the laser and of the voxel are aligned, thus, two situations may affect the quality of the part: a region beyond the target voxel may be cured, leading to an outer area A_{out}, and part of the target voxel may lack the cure, leading to an inner area A_{in}. Both are illustrated in Figure 1.3a.

The mean values of A_{in} and A_{out} were calculated for the same case study. Standard lasers for stereolithography have a radius of 0.05 millimeters, so, taking into account the considered unit of measurement, the laser has radius 0.25 units. Figure 1.3b shows the mean value of the percentage relative to the voxel area of A_{in} and A_{out}, respectively μ_{in} and μ_{out}. In all cases an area of voxels is left to cure and for some of them there is also an area reached outside the voxels. The mean value of A_{in} was above 60% for all tests. The main reasons are the assumption that the laser reaches only the center of voxels and considering voxels whose sides are twice the diameter of the laser beam. Working with smaller voxels would result in a reduction of this area, but would increase

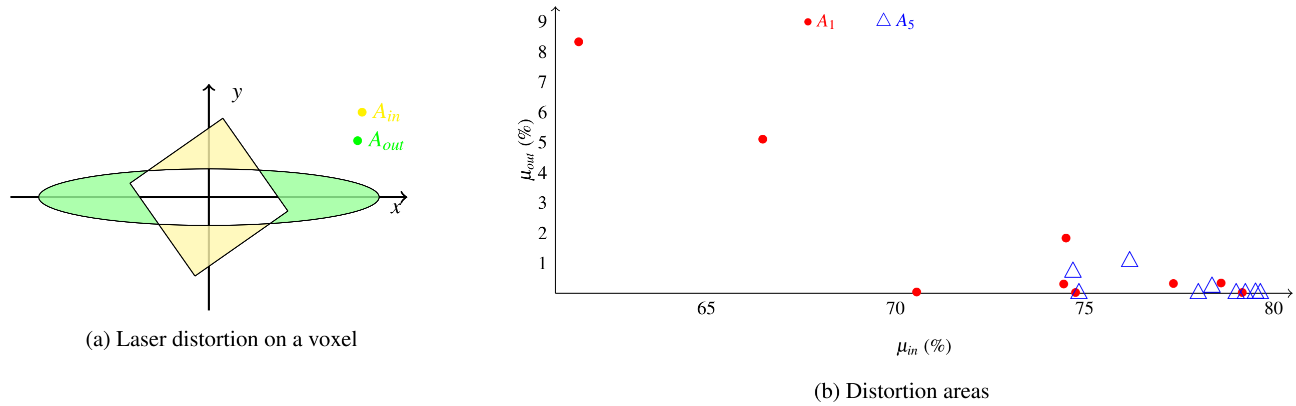


Fig. 1.3: Quality of the printed part

the values of A_{out}. The area A_{out} is almost null for most of the cases. The instance with the highest values of A_{out} corresponds to approach A_1 when applied to test T3.

The polymer at a given voxel may be affected by a beam pointing at neighbor voxels. Likewise, only the outer area of voxels in the border is relevant for the quality of the part. Therefore, the expressions of A_{in} and A_{out} are only estimate measures for the printing quality. Additionally, current 3D printing processes include a post-printing finishing phase where all part is exposed to UV light to cure any liquid resin left. This can reduce the theoretical values of A_{in} and allows to improve the produced part.

1.5 Conclusions

This work addressed the bimaterial 3D printing problem based on the instalation of galvanometer scanners on the walls of a printer. The problem is treated as locating the emitters and assigning them with the voxels of a given part, which were formulated and tested for a case study. The software CPLEX was able to find exact solutions for the considered instances. Nevertheless, these are computationally hard problems, thus heuristics should be designed to prevent cases for which this does not happen or when no commercial specialized software is available. Moreover, the unexpectedly cured area of the extreme solutions of the EAP was relatively small, while the uncured area of the part seems fairly high. In practice this latter issue can be addressed with a post-printing finishing phase, a standard 3D printing procedure. Our model can also restrict the emitter positions having in mind to reduce the laser distortion, although that may compromise the full printing of the part. Finally, the presented approach can still be used to print parts with more than 2 materials, by handling the product of the first print as the inner structure of the next one.

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