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MOORE CATEGORIES

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ABSTRACT: In 1970, M. Gerstenhaber introduced a list of axioms defining Moore categories in order to develop the Baer Extension Theory. In this paper, we study the independence of the axioms and compare them with more recent notions, showing that a Moore category is exactly a pointed, strongly protomodular and Barr-exact category with cokernels.

Introduction

For several years many category theorists were focused on defining an axiomatic context that would reflect the properties of groups and rings as the abelian categories do for abelian groups and modules. The difficulty found in weakening the axioms of abelian categories contributed for the arising of many different approaches, from the 1950's through the 1970's. Some were designed to represent a good context for non-abelian homology, such as Moore categories, while others were developed to capture more or less algebraic properties, such as Barr-exact Maltsev categories. So, unlike the abelian case, there was no outstanding theory that could be considered as a "good" generalization of groups.

In [3], D. Bourn introduced the notion of a protomodular category, whose outstanding example is the category of groups. Later in [5], he defined the notion of strong protomodular categories which capture some more group-like properties. Also based on protomodular categories, the new concept of semi-abelian categories appeared in [10], by G. Janelidze, L. Márki and W. Tholen. At this time the "old" complicated axioms from the earlier years were compared with more recent notions, establishing the existence of many disguised similarities.

Since recent notions had already appeared much earlier, although in disguised forms, we are interested in analyzing Moore categories. This notion was introduced in [9] by M. Gerstenhaber as a category suitable for developing the Baer Extension Theory. Having in mind the next higher cohomology group, containing the obstructions to extensions problems, he was concerned

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in giving a good context for the cohomology of groups. For these reasons, the categorical setting given is based on the categories of groups and rings. Like most of the theories of this time, the definition of a Moore category is also given by long list of entangled axioms. But, after the successful comparison of the "old" and "new" theories done for the semi-abelian categories, we expect to achieve some simplifications by translating the old axioms into more recent notions and discarding the needless axioms by studying their dependencies. As shown in the sequel, a Moore category turns out to be a pointed, strongly protomodular and Barr-exact category with cokernels. We conclude by presenting other examples of Moore categories besides the categories of groups and rings.

1. Moore Categories

We denote kernels by $\rightarrowtail$ and cokernels by $\twoheadrightarrow$. The notion of a *normal monomorphism* will be used in the sense of protomodularity, i.e. with respect to an equivalence relation (Section 3). We write $k \dashv R$ when a monomorphism k is normal to an equivalence relation R. The kernel equivalence relation of a morphism f is represented by $R[f]$. A short exact sequence $A \overset{k}{\rightarrowtail} B \overset{p}{\twoheadrightarrow} Q$ will also be called an *extension* (of Q by A). Moreover, if $p \cdot s = 1_Q$, then we call the diagram $A \overset{k}{\rightarrowtail} B \underset{s}{\overset{p}{\rightleftarrows}} Q$ a *split extension.*

The notion of a Moore category introduced in [9] is the following:

Definition 1.1. A category $\mathcal{C}$ with zero object, kernels and cokernels is called a *Moore category* if the following axioms hold:

axiom 1.1 3×3 lemma;
axiom 1.2 In the commutative diagram

$$\begin{array}{ccccc} A & \overset{k}{\rightarrowtail} & B & \overset{p}{\twoheadrightarrow} & Q \\ {\scriptstyle\alpha}\downarrow & & \downarrow{\scriptstyle\beta} & & \downarrow{\scriptstyle\gamma}\,\cong \\ A' & \underset{k'}{\rightarrowtail} & B' & \underset{p'}{\twoheadrightarrow} & Q' \end{array}$$

where both rows are extensions, the composition $k' \cdot \alpha$ is a kernel;
axiom 2.1 $\mathcal{C}$ has pullbacks;
axiom 2.2 Cokernels are preserved by pullbacks;
axiom 2.3 The intersection of kernels is a kernel;

axiom 3 Given a split extension $A \rightarrowtail^{k} B \underset{s}{\overset{p}{\rightleftarrows}} Q$, the pair of morphisms (k, s) is jointly epic;

axiom 4.1 There is a representative set under the equivalence relation for the extensions of Q by A;

axiom 4.2 $\mathrm{Sub}(A)$ is a set.

We first note that a Moore category has finite limits, since it is pointed and has pullbacks. So it is unnecessary to refer the kernels in the beginning of the definition.

In the next sections we are focused on proving that a Moore category is a pointed, strongly protomodular and Barr-exact category with cokernels. During this process we shall see that axioms 2.3 and 3 are redundant.

2. Protomodularity

We will start by analyzing the properties obtained from the 3×3 lemma.

Definition 2.1. A category $\mathcal{C}$ with pullbacks is called *protomodular* if the change of base functors, with respect to the fibration

$$\begin{array}{rccl} \pi : & \mathrm{Pt}\mathcal{C} & \longrightarrow & \mathcal{C}, \\ & B \underset{s}{\overset{p}{\rightleftarrows}} Q & \longmapsto & Q \quad \text{where } p \cdot s = 1_Q \end{array}$$

are conservative.

There are several alternative definitions for protomodularity. We will use the following three:

Proposition 2.2. (Proposition 7 of [3].) *A category with pullbacks is protomodular if and only if the* **pullback cancellation property** *holds: for every commutative diagram with* $p \cdot s = 1_Q$

$$\begin{array}{ccccc} A & \longrightarrow & B & \longrightarrow & C \\ \downarrow & 1 & p\downarrow\uparrow s & 2 & \downarrow \\ P & \longrightarrow & Q & \longrightarrow & R, \end{array}$$

if $\boxed{1}$ *and* $\boxed{1|2}$ *are pullbacks, then* $\boxed{2}$ *is a pullback.*

When $\mathcal{C}$ is pointed and has pullbacks, protomodularity can also be characterized by the **split short five lemma:** given a commutative diagram where the rows are split extensions,

$$\begin{array}{ccccc} A & \overset{k}{\rightarrowtail} & B & \underset{s}{\overset{p}{\rightleftarrows}} & Q \\ {\scriptstyle\alpha}\,\downarrow\cong & & \downarrow{\scriptstyle\beta} & & \cong\downarrow\,{\scriptstyle\gamma} \\ A' & \underset{k'}{\rightarrowtail} & B' & \underset{t}{\overset{p'}{\rightleftarrows}} & Q', \end{array} \qquad (1)$$

if α and γ are isomorphisms, then β is an isomorphism (Theorem 2.3 (h) of [4].).

A pair of morphisms $(x : X \longrightarrow A, y : Y \longrightarrow A)$ is *jointly strongly epimorphic* whenever a monomorphism $j : J \rightarrowtail A$ is an isomorphism provided that its pullbacks along x and y are isomorphisms. In the presence of equalizers, this notion implies that (x, y) is jointly epimorphic.

Lemma 2.3. (see [7], p. 781.) *A category with pullbacks of split epimorphisms is protomodular if and on only if for every pullback diagram with* $p \cdot s = 1_Q$

$$\begin{array}{ccc} A & \overset{a}{\longrightarrow} & B \\ \downarrow & \lrcorner & {\scriptstyle p}\downarrow\uparrow{\scriptstyle s} \\ P & \longrightarrow & Q, \end{array}$$

the pair (a, s) *is jointly strongly epimorphic.*

Proposition 2.4. *A pointed category satisfying axioms 2.1 and 1.1 is protomodular.*

Proof: Consider the diagram (1) and apply the 3×3 lemma to the commutative diagram

$$\begin{array}{ccccc} 0 & = & 0 & = & 0 \\ {\scriptstyle 0_A}\downarrow & & \downarrow{\scriptstyle 0_B} & & \downarrow{\scriptstyle 0_Q} \\ A & \overset{k}{\rightarrowtail} & B & \overset{p}{\twoheadrightarrow} & Q \\ {\scriptstyle\alpha}\,\downarrow\cong & & \downarrow{\scriptstyle\beta} & & \cong\downarrow\,{\scriptstyle\gamma} \\ A' & \underset{k'}{\rightarrowtail} & B' & \underset{p'}{\twoheadrightarrow} & Q' \end{array}$$

to conclude that the second column is an extension, thus β is an isomorphism. □

At this point we may identify regular epis with cokernels, one of the well known properties of pointed protomodular categories (Corollary 14 of [3].).

Remark 2.5. A pointed category satisfying axioms 2.1 and 1.1 also satisfies axiom 3, by Lemma 2.3 for $P = 0$.

3. Barr-exactness

We will use some of the properties of normal monos in pointed protomodular categories to prove the Barr-exactness of a Moore category.

We say that a morphism $f : X \longrightarrow A$ is *normal* to an equivalence relation $(r_1 \quad r_2) : R \rightarrowtail A \times A$ if X is an equivalence class of R, i.e. if there exists a morphism $\tilde{f}$ such that the first diagram commutes and the second is a pullback:

$$\begin{array}{ccc} X \times X & \xrightarrow{\tilde{f}} & R \\ \| & & \downarrow \\ X \times X & \xrightarrow[f \times f]{} & A \times A \end{array} \qquad\qquad \begin{array}{ccc} X \times X & \xrightarrow{\tilde{f}} & R \\ {\scriptstyle \pi}\downarrow & \lrcorner & \downarrow{\scriptstyle r_1} \\ X & \xrightarrow[f]{} & A. \end{array}$$

A normal morphism is always a monomorphism (Lemma 1 of [6].) and it is a kernel if and only if $R = R[g]$, for some morphism g (Proposition 4 of [6].).

Theorem 3.1. (Theorem 11 of [6].) *In a pointed protomodular category, if $(X \overset{x}{\rightarrowtail} A) \dashv R$ and $(Y \overset{y}{\rightarrowtail} A) \dashv S$ such that $X \wedge Y = 0$, then there is a unique morphism $\gamma : X \times Y \longrightarrow A$ with $\gamma \cdot (1 \quad 0) = x$ and $\gamma \cdot (0 \quad 1) = y$. Moreover, for the double relation $R \square S$ we have an equivalence relation*

$$\begin{array}{ccc} R\square S & \longrightarrow & A \times A \\ \begin{array}{ccc} x & R & y \\ S & & S \\ z & R & w \end{array} & \longmapsto & (x, w) \end{array}$$

such that $RS = SR = R \vee S = R \square S$ and $\gamma \dashv (R \square S)$.

Lemma 3.2. *In a pointed category satisfying axioms 2.1 and 1.1, given a commutative diagram*

$$\begin{array}{ccccc} 0 & \rightarrowtail & Y & = & Y \\ \downarrow & \lrcorner \; 1 & \downarrow{\scriptstyle y} & & \downarrow{\scriptstyle \overline{y}} \\ X & \overset{x}{\rightarrowtail} & A & \overset{r}{\twoheadrightarrow} & B \\ \| & & \downarrow{\scriptstyle s} & 2 \; \ulcorner & \downarrow{\scriptstyle \overline{s}} \\ X & \xrightarrow[\overline{x}]{} & C & \xrightarrow[\overline{r}]{} & D \end{array}$$

where the first and second lines are extensions, $\boxed{1}$ *is a pullback and* $\boxed{2}$ *is a pushout, then all lines are extensions. Moreover, there exists a unique morphism* $\gamma : X \times Y \longrightarrow A$ *such that* $\gamma \cdot (1 \quad 0) = x$*,* $\gamma \cdot (0 \quad 1) = y$ *and* $X \times Y \overset{\gamma}{\rightarrowtriangle} A \xrightarrow{\overline{r}\cdot s} D$ *is an extension.*

Proof: We have $x \dashv R[r]$, $y \dashv R[s]$ such that $X \wedge Y = 0$. Applying Theorem 3.1, there exists $\gamma : X \times Y \rightarrowtail A$ such that $\gamma \cdot (1 \quad 0) = x$, $\gamma \cdot (0 \quad 1) = y$ and $\gamma \dashv (R[r] \vee R[s])$. Since $\boxed{2}$ is a pushout, we have $R[r] \vee R[s] = R[t]$, for the cokernel $t = \overline{r} \cdot s$, thus $\gamma = \ker(t)$. Applying the 3×3 lemma to the commutative diagram

$$\begin{array}{ccccc}
Y & \xrightarrow{(0 \quad 1)} & X \times Y & \xrightarrow{\pi} & X \\
\| & & \downarrow \gamma & & \downarrow \overline{x} \\
Y & \xrightarrow{y} & A & \xrightarrow{s} & C \\
\downarrow & & \downarrow t & & \downarrow \overline{r} \\
0 & \longrightarrow & D & = & D
\end{array}$$

where all rows, the first and second columns are extensions and the third column is a zero sequence, we prove that $X \xrightarrow{\overline{x}} C \xrightarrow{\overline{s}} D$ is an extension. Finally, we conclude that $Y \xrightarrow{\overline{y}} B \xrightarrow{\overline{s}} D$ is an extension by applying the 3×3 lemma to the original diagram. $\square$

Lemma 3.3. *In a pointed category with cokernels satisfying axioms 2.1 and 1.1, any monomorphism* $\overline{y}$ *that factorizes as* $\overline{y} = r \cdot y$*, with* r *a cokernel and* y *a kernel, is also a kernel.*

Proof: Consider $x = \ker(r)$, $s = \text{coker}(y)$ and $\overline{s} = \text{coker}(\overline{y})$. From $\overline{s} \cdot r \cdot y = 0$, we get a unique morphism $\overline{r}$ such that $\overline{r} \cdot s = \overline{s} \cdot r$. We have the diagram of Lemma 3.2 with $\boxed{1}$ a pullback, since $\overline{y}$ is a monomorphism, and $\boxed{2}$ a pushout, since $\overline{s} = \text{coker}(\overline{y})$. $\square$

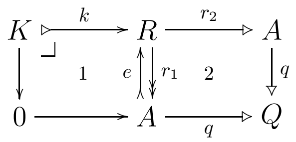
Proposition 3.4. *In a pointed category with cokernels satisfying axioms 2.1 and 1.1, reflexive relations are effective equivalence relations.*

Proof: Suppose $(r_1 \quad r_2) : R \rightarrowtail A \times A$ is a reflexive relation, i.e. there exists a monomorphism $e : A \rightarrowtail R$ such that $r_1 \cdot e = r_2 \cdot e = 1_A$. For $k = \ker(r_1)$, the pair (k, e) is jointly epic.

The composition $r_2 \cdot k$ is a monomorphism, since $r_2 \cdot k \cdot x = r_2 \cdot k \cdot y$ implies $(r_1 \quad r_2) \cdot k \cdot x = (r_1 \quad r_2) \cdot k \cdot y$, allowing us to conclude that $x = y$. This monomorphism $r_2 \cdot k$ factors by a cokernel and a kernel, thus by Lemma 3.3 is also a kernel. Consider $q = \mathrm{coker}(r_2 \cdot k)$. We have $q \cdot r_1 = q \cdot r_2$ by the fact that (k, e) is jointly epic and

$$q \cdot r_1 \cdot k = 0 = q \cdot r_2 \cdot k$$
$$q \cdot r_1 \cdot e = q = q \cdot r_2 \cdot e.$$

Applying the pullback cancellation property of protomodular categories to the commutative diagram



we conclude that $\boxed{2}$ is a pullback. □

As a consequence, we may identify kernels with normal monomorphisms.

Remark 3.5. A pointed category with cokernels satisfying axioms 2.1 and 1.1 also satisfies axiom 2.3, since the stability for intersections holds for normal monomorphisms ($f \dashv R$ and $g \dashv S$, imply that $(f \wedge g) \dashv (R \wedge S)$).

To prove that a Moore category is regular, we will use the pullback stability of axiom 2.2.

Proposition 3.6. *A pointed category with cokernels satisfying axioms 2.1 and 1.1 is protomodular and regular.*

Proof: Based on Proposition 3.2 of [10], if $\mathcal{C}$ is a pointed category with kernels, cokernels of kernels, pullback-stable cokernels and such that $\ker(f) = 0$ iff f is a monomorphism, then $\mathcal{C}$ has a pullback-stable (cokernel, mono)-factorization system. □

Conversely, a pointed protomodular and regular category satisfies the 3×3 lemma (Theorem 12 of [7]) and the pullback-stability holds for cokernels, since they coincide with the regular epimorphisms.

Proposition 3.7. *A pointed category with cokernels satisfying axioms 2.1 and 1.1 is protomodular and Barr-exact.*

Proof: By Propositions 3.6 and 3.4. □

4. Strong Protomodularity

Axiom 1.2 is finally used in order to establish strong protomodularity.

Definition 4.1. A category $\mathcal{C}$ with finite limits is called *strongly protomodular* if the change of base functors, with respect to the fibration $\pi : \mathrm{Pt}\mathcal{C} \longrightarrow \mathcal{C}$, are left exact, conservative and reflect normal monomorphisms.

We have seen that normal monomorphisms in $\mathcal{C}$ are kernels. Normal monomorphisms in the category of pointed objects have the following characterization:

Proposition 4.2. (Proposition 2.1 of [5].) *When a category $\mathcal{C}$ is quasi-pointed and protomodular, a map*

$$\begin{array}{ccc} X & \xrightarrow{\;j\;} & Y \\ & {\scriptstyle f}\searrow \quad \swarrow{\scriptstyle g} & \\ & Q & \end{array}$$

is a normal monomorphism in $\mathcal{C}/Q$ if and only if $j \cdot \ker(f)$ is a normal monomorphism in $\mathcal{C}$. The same result holds in $\mathrm{Pt}\mathcal{C}[Q]$.

So, j is a normal monomorphism in $\mathcal{C}/Q$ (or $\mathrm{Pt}\mathcal{C}[Q]$) if and only if $j \cdot \ker(f)$ is a kernel in $\mathcal{C}$.

Proposition 4.3. *A pointed category with cokernels satisfying axioms 2.1, 1.1 and 1.2 is strongly protomodular.*

Proof: Since the category is pointed and protomodular, it suffices to prove that the change of base functors

$$\begin{array}{ccccc} (0_Q)^* : & \mathrm{Pt}\mathcal{C}[Q] & \longrightarrow & \mathcal{C} \\ & B \underset{s}{\overset{p}{\rightleftarrows}} Q & \longmapsto & A = \ker(p) \\ & {\scriptstyle\beta}\downarrow \qquad \| & & \downarrow{\scriptstyle\alpha} \\ & B' \underset{s'}{\overset{p'}{\rightleftarrows}} Q & & A' = \ker(p') \end{array}$$

reflect normal monomorphisms. This is given by axiom 1.2. □

Conversely, we have:

Proposition 4.4. *A pointed, strongly protomodular and Barr-exact category with cokernels satisfies axiom 1.2.*

Proof: The diagram of axiom 1.2 induces the commutative diagram

$$
\begin{array}{ccccc}
A & \xrightarrow{(0 \quad k)_Q} & R[p] & \underset{(1_B \quad 1_B)_Q}{\overset{p_1}{\rightleftarrows}} & B \\
{\scriptstyle \alpha}\big\downarrow & & {\scriptstyle 1_A \times_Q \beta}\big\downarrow & & \big\| \\
A' & \xrightarrow[(0 \quad k')_Q]{} & B \times_Q B' & \underset{(1_B \quad \beta)_Q}{\overset{\pi_B}{\rightleftarrows}} & B.
\end{array}
$$

Since the category is strongly protomodular and α is a kernel, the composite $(0 \quad k')_Q \cdot \alpha = 1_A \times_Q \beta \cdot (0 \quad k)_Q$ is a kernel. By Lemma 3.3 applied to the monomorphism $k' \cdot \alpha = \pi_{B'} \cdot ((0 \quad k')_Q \cdot \alpha)$ is a kernel. □

Our results can be gathered into the following characterization:

Theorem 4.5. *A Moore category is a pointed, strongly protomodular and Barr-exact category with cokernels.*

5. Mo(o)re Examples

(1) Grp, Rng.

(2) Abelian categories. They are pointed and have cokernels by definition. They are also essentially affine, since they are additive and have finite limits (Corollary 5 of [3]), thus strongly protomodular ([5]). Finally, they are Barr-exact by the Tierney equation.

(3) Heyting semilattices. It is well known that the category $\mathcal{H}$ of Heyting semilattices (meet-semilattices with implication) is semi-abelian.

Before proving the strong protomodularity of $\mathcal{H}$, we first note that a morphism $\alpha : A \longrightarrow A'$ is a normal monomorphism in $\mathcal{H}$ if and only if $\alpha(A)$ is a filter in A'. In fact, if $\alpha \dashv R$, then $(\alpha(a_1), \alpha(a_2)) \in R$, for every pair of elements (a_1, a_2) of A, and $(\alpha(a), a') \in R$ implies $a' \in \alpha(A)$. Consequently, $(1_{A'}, a') \in R$ if and only if $a' \in \alpha(A)$. Now $1_{A'} = \alpha(1_A)$, α is closed for meets and for $a \in A, a' \in A'$ such that $\alpha(a) \leq a'$, we have

$$
\begin{aligned}
(\alpha(a), 1_{A'}), (a', a') \in R \ &\Rightarrow (\alpha(a) \to a', 1_{A'} \to a') \in R \\
&\Rightarrow (1_{A'}, a') \in R \\
&\Rightarrow a' \in \alpha(A),
\end{aligned}
$$

proving that $\alpha(A)$ is a filter in A'. Conversely, if $\alpha(A)$ is a filter in A', then $S = \{(a'_1, a'_2) \in A' \times A' : a'_1 \to a'_2, a'_2 \to a'_1 \in \alpha(A)\}$ is an equivalence relation on A'. Moreover, $(1_{A'}, a') \in S$ if and only if $a' \in \alpha(A)$ and $(\alpha(a_1), \alpha(a_2)) \in S$, for every pair (a_1, a_2) of elements of

A, since α preserves the implication. Hence, there exists a morphism $\tilde{\alpha} : A \times A \longrightarrow S$, with $\tilde{\alpha}(a_1, a_2) = (\alpha(a_1), \alpha(a_2))$, such that $\alpha \dashv S$.

For any Heyting semilattice Q, we must prove that the change of base functor of Proposition 4.3 reflects normal monomorphisms. Note that $A = \{b \in B : p(b) = 1_Q\}$, $k : A \hookrightarrow B$ is an inclusion and $\alpha(a) = \beta(a)$, for every element a of A. Suppose $\alpha(A) = \beta(A)$ is a filter in A'. We want to prove that $\beta \cdot k(A) = \beta(A)$ is a filter in B'. Consider $a \in A, b' \in B'$ such that $\beta(a) \leq b'$. Then

$$\begin{aligned} \beta(a) \leq b' &\Rightarrow \beta(a) \to b' = 1_{B'} \\ &\Rightarrow p'(\beta(a) \to b') = 1_Q \\ &\Rightarrow 1_Q \to p'(b') = 1_Q \\ &\Rightarrow p'(b') = 1_Q \\ &\Rightarrow b' \in A'. \end{aligned}$$

Since $\beta(A)$ is a filter in A' and $\beta(a) \leq b'$, with $b' \in A'$, then $b' \in \beta(A)$.

(4) Grp($\mathcal{C}$), Rng($\mathcal{C}$), for an elementary topos $\mathcal{C}$ with a natural number object. They are obviously pointed and the existence of cokernels is guaranteed by the natural number object. When $\mathcal{C}$ is finitely complete, the categories of internal groups and rings are strongly protomodular. They are Barr-exact because $\mathcal{C}$ is also Barr-exact.

(5) Pt$\mathcal{C}[Q]$, for $\mathcal{C}$ strongly protomodular, Barr-exact with coequalizers. They are obviously pointed and coequalizers in $\mathcal{C}$ give cokernels in the category of pointed objects. Finally, since $\mathcal{C}$ is strongly protomodular and Barr-exact, the same holds for Pt$\mathcal{C}[Q]$.

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