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IMAGE CONTRAST ENHANCEMENT USING SPLIT BREGMAN METHOD

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ABSTRACT: In this paper we propose a variational method for image contrast enhancement, by keeping the image details and correcting the non-uniform illumination. It is a minimization problem, where the objective functional consists of two different fitting terms: a L^1 term that matches the gradients of the input and reconstructed images, for preserving the image details, and a L^2 term that measures the misfit between the reconstructed image and the mean value of the input image, for reducing the image variance and thus correcting the illumination. For solving this minimization problem we apply the split Bregman method, which is an efficient and fast iterative method suitable for this type of non-differentiable and convex minimization problem, involving a L^1 term. Some experimental results show the effectiveness of the method.

KEYWORDS: Variational enhancement model, Split Bregman method.

1. Introduction

Image contrast enhancement is an image processing technique, whose purpose is to improve the image quality, for human interpretation of the image contents or for supplying a good input in automated image processing systems.

In the literature there exists a plethora of contrast enhancement methods, which are based, for example, on histogram equalization, edge enhancement, edge sharpening, filtering and restoration. For a detailed description of these type of methods see for example the book [7]. In this paper we focus on a particular variational PDE (partial differential equation) approach for contrast enhancement. We refer for example to the book [1] for an overview of the application of functional analysis techniques and the theory of partial differential equations to different image processing problems, such as restoration of degraded images, denoising, segmentation, inpainting, decomposition into cartoon and texture, optical flow and image classification.

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In the variational approach, used herein, the objective is to minimize an appropriate energy (or functional), whose corresponding Euler-Lagrange equation involves a PDE, that can be afterwards solved by a suitable computational method. Our energy functional is a modification of that proposed in [8]. It is composed of two fitting terms. One fits the gradients of the input and reconstructed images, it is measured with the L^1-norm, and aims at preserving image details. The L^1-norm is the chosen measure because it has the property of better preserving discontinuities, when compared to the L^2-norm. The second term fits the reconstructed image with the mean value of the input image, in the L^2-norm, for correcting non-uniform illumination. Due to its particular structure, we then choose the split Bregman method [5] to solve this variational problem. This is a particular efficient iterative method applicable to a wide class of $L1$-regularized optimization problems.

After this introduction the rest of the paper includes the description of the variational problem in Section 2, its numerical solution in Section 3, some applications in Section 4 and finally the paper ends with some conclusions and comments.

2. Description of the model

In [8] it is proposed the following variational model for contrast enhancement

$$\begin{aligned}
&\min_u \Big\{ \int_\Omega |\nabla u - \nabla f|^2 dx + \lambda \int_\Omega (u - \tilde{u})^2 dx \Big\} \\
&= \min_u \Big\{ \|\nabla u - \nabla f\|^2_{L^2(\Omega)} + \lambda \|u - \tilde{u}\|^2_{L^2(\Omega)} \Big\},
\end{aligned} \tag{1}$$

where $f : \Omega \to R$ is the original (grayscale) input image, $\Omega \subset R^2$ represents the image pixel domain, x is a point (*i.e.* a pixel) in Ω, ∇ denotes the gradient operator, $|.|$ is the Euclidean norm in R^2, $\tilde{u}$ is the mean value of the reconstructed image $u : \Omega \to R$, $L^2(\Omega)$ is the space of square integrable functions in Ω, and $\|.\|_{L^2(\Omega)}$ denotes the L^2-norm. This model contains two quadratic fitting terms. The first fits the gradients of u and f, and consequently aims at preserving image details. The second term intends to reduce the effect of nonuniform illumination by fitting u and its mean value $\tilde{u}$, thus by decreasing the image variance. The parameter λ is a positive constant that balances the influence of the two fitting terms. After replacing $\tilde{u}$ by $\tilde{f}$, the mean value of the input image f, it is shown that the problem does not depend on the value $\tilde{f}$, and the problem is solved, for $\tilde{f} = 0$ in the Fourier domain, using the discrete Fourier transform.

In this paper we modify the model (1) by replacing the L^2-norm by the L^1-norm for measuring the misfit between ∇u and ∇f and we also replace the mean value $\tilde{u}$ by $\tilde{f}$. This yields the following variational model

$$\begin{aligned} &\min_u \left\{ \int_\Omega |\nabla u - \nabla f| dx + \frac{\lambda}{2} \int_\Omega (u - \tilde{f})^2 dx \right\} \\ &= \min_u \left\{ \|\nabla u - \nabla f\|_{L^1(\Omega)} + \frac{\lambda}{2} \|u - \tilde{f}\|^2_{L^2(\Omega)} \right\} \end{aligned} \tag{2}$$

where $L^1(\Omega)$ is the space of absolutely integrable functions in Ω and $\|.\|_{L^1(\Omega)}$ denotes the L^1-norm. The reason for replacing the L^2-norm by the L^1-norm is related to the fact that the L^2-norm of the gradient tends to smear image discontinuities, as opposed to the L^1-norm that tends to preserve the discontinuities, which in image processing corresponds to sharp edges. In addition to this advantage, the presence of the L^1 fitting term in (2) permits the use of fast and effective algorithms for computing its solution. In effect, problem (2) belongs to the general class of L^1-regularized problems of the form

$$\min_u \left\{ \|\phi(u)\|_{L^1(\Omega)} + H(u) \right\}$$

where both $\|\phi(u)\|_{L^1(\Omega)}$ and $H(u)$ are convex functions. This kind of models can be efficiently solved with the split Bregman method of [5]. This is an appropriate algorithm for solving non-differentiable convex minimization problems, involving L^1 or TV (total variation) terms. We refer to [3, 4, 6, 9] for a few examples of different applications of the method.

In our case $\phi(u) = \nabla u - \nabla f$ and $H(u) = \lambda \|u - \tilde{f}\|_{L^2(\Omega)}$, and in the next section we apply split Bregman method to solve (2).

3. Numerical solution based on split Bregman method

A critical and first aspect of the split Bregman method is the separation of the L^1 and L^2 terms, which is achieved by introducing an auxiliary variable. Thus, we first replace (2) by the following constrained optimization problem

$$\min_u \left\{ \|d\|_{L^1(\Omega)} + \frac{\lambda}{2} \|u - \tilde{f}\|_{L^2(\Omega)} \right\}$$
$$\text{subject to} \quad d = \nabla u - \nabla f,$$

and then reformulate it as an unconstrained problem, by introducing a quadratic penalty function, that is

$$\begin{aligned} \min_u \Big\{ \|d\|_{L^1(\Omega)} + \frac{\lambda}{2} \|u - \tilde{f}\|^2_{L^2(\Omega)} \\ + \frac{\alpha}{2} \|d - (\nabla u - \nabla f)\|^2_{L^2(\Omega)} \Big\}. \end{aligned} \tag{3}$$

Then, the split Bregman method consists in solving the following sequence of problems for $k = 0, 1, 2, \ldots$

$$\begin{cases} (u^{k+1}, d^{k+1}) = \\ arg \min_{d,u} \Big\{ \|d\|_{L^1(\Omega)} + \frac{\lambda}{2} \|u - \tilde{f}\|^2_{L^2(\Omega)} \\ \qquad + \frac{\alpha}{2} \|d - (\nabla u - \nabla f) - b^k\|^2_{L^2(\Omega)} \Big\}, \\ \\ b^{k+1} = b^k + \nabla u^{k+1} - \nabla f - d^{k+1}. \end{cases} \tag{4}$$

where the new vector b^k results from the Bregman iteration [2], that is a strategy for enforcing the constraint $d = \nabla u - \nabla f$, using a fixed penalty parameter α. This strategy is an alternative to the conventional continuation technique to solve (3) with an increasing sequence of penalty parameters $\alpha_1 < \alpha_2 < \ldots < \alpha_n$ tending to ∞, for accurately enforcing the constraint. The minimization problem in (4) is solved by iteratively minimizing with respect u and d, alternatively, which means the following two steps are performed.

Step 1- Minimization with respect to u (with d fixed)

$$\begin{aligned} u^{k+1} = arg \min_u \Big\{ \frac{\lambda}{2} \|u - \tilde{f}\|^2_{L^2(\Omega)} \\ + \frac{\alpha}{2} \|d^k - (\nabla u - \nabla f) - b^k\|^2_{L^2(\Omega)} \Big\}, \end{aligned} \tag{5}$$

for which the optimality condition (derived from the Euler-Lagrange equation) is, in Ω

$$(\lambda - \alpha\Delta) u^{k+1} = \lambda \tilde{f} - \alpha \operatorname{div}(d^k + \nabla f - b^k),$$

with Δ and div denoting the Laplace and divergence operators, respectively, along with the non-homogeneous Neumann boundary condition on the boundary $\partial\Omega$ of Ω

$$\frac{\partial u^{k+1}}{\partial n} = (d^k - \nabla f - b^k) \cdot n,$$

where "$\cdot$" denotes the inner product in R^2 and n is the unit outward normal to $\partial\Omega$.

This problem can be solved efficiently with the Gauss-Seidel method, since the system is diagonally dominant. The solution $u_{i,j}^{k+1}$ at each pixel (i,j) in Ω (excepting in $\partial\Omega$) is defined by

$$\begin{aligned} u_{i,j}^{k+1} &= \frac{1}{\lambda + 4\alpha}\left[\alpha U_{i,j} + \lambda \tilde{f}_{i,j} - \alpha v_{i,j}^k\right] \\ U = (U_{i,j}) &:= \left(u_{i-1,j}^{k+1} + u_{i+1,j}^k + u_{i,j-1}^{k+1} + u_{i,j+1}^k\right) \\ v^k = (v_{i,j}^k) &:= (\mathrm{div}(d^k + \nabla f - b^k))_{i,j}. \end{aligned}$$

Here we use finite differences for approximating the derivatives in the gradient ∇ and divergence div operators, respectively. In particular the discretization used for $v_{i,j}^k$ is obtained by applying backward finite differences for divd^k and divb^k and centered finite differences for Δf

$$\begin{aligned} v_{i,j}^k = 2d_{i,j}^k - d_{i-1,j}^k - d_{i,j-1}^k - 2b_{i,j}^k + b_{i-1,j}^k + b_{i,j-1}^k \\ + f_{i-1,j}^k + f_{i+1,j}^k + f_{i,j-1}^k + f_{i,j+1}^k - 4f_{i,j}^k. \end{aligned}$$

The Neumann boundary condition is implicitly imposed in $\partial\Omega$, the boundary of the rectangular pixel domain, by using backward finite differences, in the right and top sides, and forward finite differences in the left and bottom sides.

Step 2- Minimization with respect to d (with u fixed)

$$\begin{aligned} d^{k+1} = arg \min_d \Big\{ & \|d\|_{L^1(\Omega)} \\ & + \frac{\alpha}{2}\|d - (\nabla u^{k+1} - \nabla f) - b^k\|_{L^2(\Omega)}^2 \Big\}. \end{aligned}$$

This problem can be explicitly solved using shrinkage operation (known as well as soft thresholding) at each pixel (i,j)

$$d_{i,j}^{k+1} = shrink\left(\left(\nabla u^{k+1} - \nabla f + b^k\right)_{i,j}, \frac{1}{\alpha}\right)$$

where for $z, \gamma \in R$

$$shrink(z, \gamma) = \frac{z}{|z|} \times \max(|z| - \gamma, 0).$$

Summarizing, the split Bregman method for model (2) is as follows:

Algorithm -

Input - Original image f.

Initialize - $u^0 = f$, $d^0 = b^0 = 0$, and fix λ, α, tol.

While $|u^k - u^{k-1}| > tol$

$u^{k+1} = \frac{1}{\lambda+4\alpha}\left[\alpha U + \lambda \tilde{f} - \alpha v^k\right], \quad \text{in } \Omega,$

$\frac{\partial u^{k+1}}{\partial n} = (d^k - \nabla f - b^k) \cdot n, \quad \text{in } \partial\Omega,$

$d^{k+1} = shrink\left(\nabla u^{k+1} - \nabla f + b^k, \frac{1}{\alpha}\right),$

$b^{k+1} = b^k + \nabla u^{k+1} - \nabla f - d^{k+1}.$

End

Output - Image u^k.

4. Applications

Some results obtained with our proposed model are shown in this section. All the experiments were implemented with the software MATLAB® R2014a (The Mathworks, Inc.)

Figure 1 shows the contrast enhancement with our method for a standard test image (a scalar image with 512×343 pixels), downloaded from the IPOL archive (http://www.ipol.im/). As this figure demonstrates, the details are kept and the dark regions become more visible in the enhanced image. In addition, and as expected, when λ increases (λ is the parameter associated with the fitting term intended to reduce the non-uniform illumination) the result tends to the mean value of the input image.

Figure 2 depicts the results of our method applied to a medical (RGB - red, green, blue) image (with 536×536 pixels), acquired with the wireless capsule Pillcam Colon 2 of *Given Imaging*. It displays a colonic polyp (the reddish region in the top left subfigure) exhibiting strong texture. We applied

FIGURE 1. Top left: Original image. Top right: $\lambda = 0.005$, $\alpha = 1$. Bottom left: $\lambda = 0.01$, $\alpha = 1$. Bottom right: $\lambda = 0.05$, $\alpha = 1$.

the algorithm independently to each channel. The original medical image (the top left subfigure) has a non-uniform illumination, with low contrast in some regions, that is corrected and enhanced with the proposed method. The influence of the model parameters (λ and α) is also illustrated in these results. Increasing λ results in an averaged image, tending to the mean value of the input image, and by increasing α the contrast enhancement is enforced.

In Figure 3 we can see the results for another medical image (with size 536×536 pixels). It is an inhomogeneous illuminated retinal fundus image, provided by the company *Retmarker* (http://www.retmarker.com/), and obtained from a patient screened according to the Diabetic Retinopathy Screening Program of Portugal. We have processed with our method the grayscale version (second column) as well as each color channel separately (first column). Again these results show the good contrast enhancement improvement achieved with our method.

5. Conclusions

In this paper we propose an inverse variational model for contrast enhancement together with the split Bregman method for its numerical solution. Applications of the proposed method to different types of images show its good performance. The model involves some parameters that are tuned and fixed

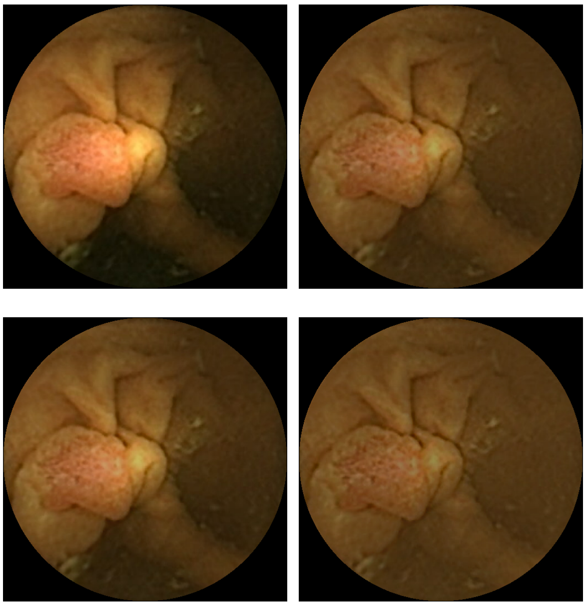

FIGURE 2. Top left: Polyp image obtained with Pillcam Colon 2, by courtesy of University Hospital of Coimbra, Portugal. Top right: $\lambda = 1$, $\alpha = 150$. Bottom left: $\lambda = 1$, $\alpha = 200$. Bottom right: $\lambda = 2$, $\alpha = 200$.

manually. In the future an automatic or self-adapting method for choosing these parameters will be studied.

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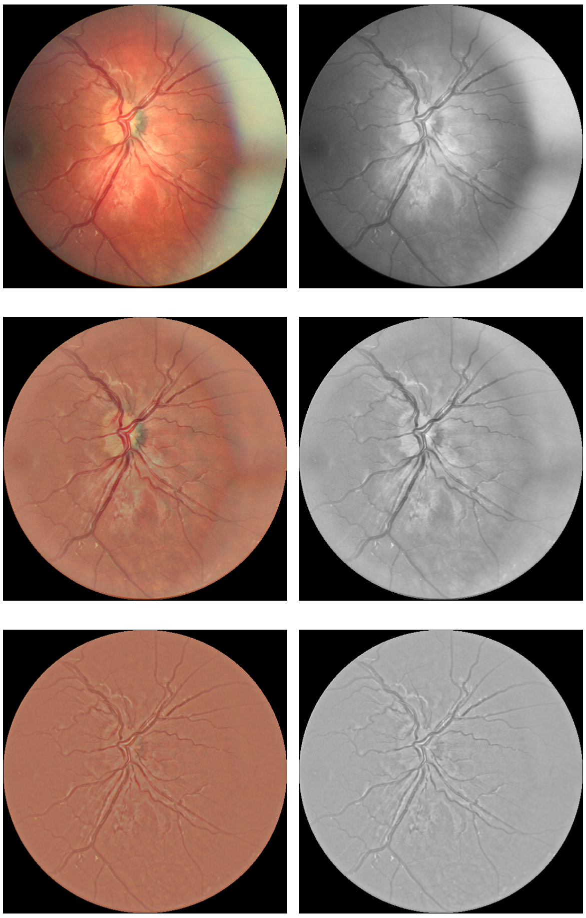

FIGURE 3. First column: Original (RGB) retinal fundus image and results with ($\lambda = 1$, $\alpha = 5$) in 2nd row and ($\lambda = 5$, $\alpha = 5$) in 3rd row. Second column: Grayscale version and results with ($\lambda = 1$, $\alpha = 5$) in 2nd row and ($\lambda = 5$, $\alpha = 5$) in 3rd row.

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