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IMAGE FUSION WITH SIMULTANEOUS CARTOON AND TEXTURE DECOMPOSITION

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Abstract: Image fusion is a technique that merges the information of multiple images, representing the same scene, to produce a single image that should gather the major and meaningful information contained in the different images. On the other hand, cartoon+texture image decomposition is another image processing technique, that decomposes the image into the sum of a cartoon image, containing the major geometric information, *i.e.*, piece-wise smooth regions, and a textural image, containing the small details and oscillating patterns. We propose a model to perform the fusion of multiple images, relying on gradient information, that provides as well a cartoon and texture decomposition of the fused image. The model is formulated as a variational minimization problem and solved with the split Bregman method. The suitability of the proposed model is illustrated with some earth observation images and also medical images.

Keywords: Variational Model, Image Fusion, Cartoon+Texture, Optimization.

1. Introduction

Image fusion is an image processing tool, that generates a single composite image from multisource images, of the same scene. It is particularly important in remote sensing (it makes possible, for example, to combine the information of earth observation images acquired by different sensors or satellites, for improving earth surveillance and monitoring) and in medicine (the fused image obtained from different medical imaging modalities may improve diagnosis and prognosis).

In this paper we propose a variational model for image fusion (we refer to [4, 6] for two related variational models) and that performs simultaneously a cartoon + texture decomposition of the fused image.

The proposed model fuses the gradients of the input images and combines this fusion with perceptual enhancement and intensity correction. At the same time it also does the decomposition of the fused image into cartoon

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and textural parts, adopting the generalized denoising model of [3], based on total variation regularization. Finally we apply the split Bregman method [5] to solve the model.

After this introduction, the rest of the paper is organized in three sections: Section 2 describes the model, Section 3 explains the application of the split Bregman method and finally in Section 4 some experimental results are shown.

2. Proposed Model

Let $I_i : \Omega \to [0,1]$, $i = 1, \ldots, N$, be (normalized) grayscale and multisource images, where $\Omega \subset R^2$ represents the pixel domain, and for a pixel $x \in \Omega$ $I_i(x)$ is the intensity of I_i at x. We define $g := \sum_{i=1}^{N} w_i \nabla I_i$, a weighted average of the gradients of I_i with weights defined by $w_i := |\nabla I_i| / \sum_{j=1}^{N} |\nabla I_j|$, where $|.|$ is the Euclidean nom and ∇ the gradient operator. We also define $I_0 := \sum_{i=1}^{N} z_i I_i$, a weighted average of the input images I_i (a possible choice for these weights is $z_i := w_i$ or $z_i := |I_i| / \sum_{j=1}^{N} |I_j|$).

In [4] the fusion model corresponds to the following minimization problem

$$\begin{aligned} &\min_I \int_\Omega \left[|\nabla I - g| + \frac{\eta}{2}|I - \frac{1}{2}|^2 + \frac{\mu}{2}|I - I_0|^2 \right] dx \\ &= \min_I \left[\|\nabla I - g\|_{L^1(\Omega)} \frac{\eta}{2} \|I - \frac{1}{2}\|^2_{L^2(\Omega)} + \frac{\mu}{2} \|I - I_0\|^2_{L^2(\Omega)} \right], \end{aligned}$$

where $L^1(\Omega)$ and $L^2(\Omega)$ are the spaces of, respectively, absolutely and square integrable functions in Ω, with norms denoted by $\|.\|_{L^1(\Omega)}$ and $\|.\|_{L^2(\Omega)}$, respectively. In this model, the fused image I is searched in such a way, that its gradient matches the combined gradient g (the function g merges the gradient information of all the different input images I_i). In addition the linear combination of fitting terms $\frac{\eta}{2}\|I - \frac{1}{2}\|^2_{L^2(\Omega)} + \frac{\mu}{2}\|I - I_0\|^2_{L^2(\Omega)}$ (as proposed in [1]) makes the fused image I to be perceptually more uniform and close to the average I_0 (the weighted sum of the several input images); η and μ are two positive parameters that balance the influence of each term.

On the other hand, in [3] the following model for image cartoon+texture decomposition is defined

$$\begin{aligned} &\min_{(u,v)} \int_\Omega \left[h(x)|\nabla u| + \frac{1}{2\theta}|u + v - f|^2 + \alpha|v| \right] dx \\ &= \min_{(u,v)} \left[TV_h(u) + \frac{1}{2\theta}\|u + v - f\|^2_{L^2(\Omega)} + \alpha\|v\|_{L^1(\Omega)} \right] \end{aligned}$$

Here $f : \Omega \subset R^2 \to R$ is the input (grayscale) image and the goal is to find a decomposition $u+v$ of f, that is $f \sim u+v$, where the image u represents the geometric information of f (*i.e.*, the cartoon or piecewise smooth regions) and the image v captures the texture information. The function h is an edge detector, that vanishes at object boundaries (such as $h(|\nabla f|) = 1/(1 + \beta|\nabla f|^2)$, with β an arbitrary positive constant), $TV_h(u)$ is the total variation norm of the function u weighted by h and $\theta > 0$ is a very small parameter, so that we almost have $f = u + v$. The terms $TV_h(u)$ and $\|v\|_{L^1(\Omega)}$ act as regularizers for u and v, respectively, and $\alpha > 0$ is a regularization parameter.

By combining the ideas of these two aforementioned models, we propose in this paper a variational model for multisource image fusion that simultaneously performs a cartoon and texture decomposition of the fused image. Given the normalized grayscale multi-source images $\{I_i\}_i^N$, the combined gradient function g and the average image I_0, already defined, we search for a fused image in the form $u+v$, as the solution of the following unconstrained optimization problem (herein denoted by $\mathcal{P}$)

$$\min_{(u,v)} \Big[\|\nabla(u+v) - g\|_{L^1(\Omega)} + TV_h(u) + \alpha\|v\|_{L^1(\Omega)} \\ + \frac{1}{2\theta}\|(u+v) - I_0\|^2_{L^2(\Omega)} + \frac{\eta}{2}\|(u+v) - \frac{1}{2}\|^2_{L^2(\Omega)} \Big].$$

where for simplicity $h = 1$.

3. Numerical Solution

Since the objective functional in $\mathcal{P}$ is convex, the solution of $\mathcal{P}$ can be computed by minimizing separately, with respect to u and v, that is by solving the cartoon and texture subproblems (denoted by C and T, respectively):

Subproblem C - v being fixed, find u solution of

$$\min_{u} \Big[\|\nabla(u+v) - g\|_{L^1(\Omega)} + TV_h(u) \\ + \frac{1}{2\theta}\|(u+v) - I_0\|^2_{L^2(\Omega)} + \frac{\eta}{2}\|(u+v) - \frac{1}{2}\|^2_{L^2(\Omega)} \Big].$$

Subproblem T - u being fixed, find v solution of

$$\min_{v} \Big[\|\nabla(u+v) - g\|_{L^1(\Omega)} + \alpha\|v\|_{L^1(\Omega)} \\ + \frac{1}{2\theta}\|(u+v) - I_0\|^2_{L^2(\Omega)} + \frac{\eta}{2}\|(u+v) - \frac{1}{2}\|^2_{L^2(\Omega)} \Big].$$

Due to the structure of subproblems C and T, involving particular L^1 and L^2 terms (that are convex functions) a suitable iterative method for approximating their solutions, and that we adopt in this paper, is the split Bregman method [5].

Firstly, extra variables are introduced through the constraints $d_1 = \nabla(u + v) - g$, $d_2 = \nabla u$, for subproblem C, and $e = \nabla(u + v) - g$, $z = v$ for subproblem T. The goal is to separate completely the L^1 and L^2 terms in each subproblem. Then we enforce these constraints with the Bregman iteration process [2]. Therefore we have a sequence of cartoon and texture subproblems, respectively C^k and T^k, for $k = 1, 2, \ldots$, as follows:

Subproblem C^k - v^k being fixed, find $(u^{k+1}, d_1^{k+1}, d_2^{k+1})$ solution of

$$\begin{cases} \min\limits_{(u,d_1,d_2)} \Big[\|d_1\|_{L^1(\Omega)} + \|d_2\|_{L^1(\Omega)} + \\ \dfrac{1}{2\theta}\|(u+v^k) - I_0\|^2_{L^2(\Omega)} + \dfrac{\eta}{2}\|(u+v^k) - \dfrac{1}{2}\|^2_{L^2(\Omega)} \\ +\dfrac{\lambda_1}{2}\|d_1 - \nabla(u+v^k) + g - b_1^k\|^2_{L^2(\Omega)} \\ +\dfrac{\lambda_2}{2}\|d_2 - \nabla u - b_2^k\|^2_{L^2(\Omega)}\Big], \end{cases}$$

$$b_1^{k+1} = b_1^k - d_1^{k+1} + \nabla(u^{k+1} + v^k) - g,$$

$$b_2^{k+1} = b_2^k - d_2^{k+1} + \nabla u^{k+1}.$$

Subproblem T^k - u^{k+1} being fixed, find $(v^{k+1}, e^{k+1}, z^{k+1})$ solution of

$$\begin{cases} \min\limits_{(v,e,z)} \Big[\|e\|_{L^1(\Omega)} + \alpha\|z\|_{L^1(\Omega)} + \\ \dfrac{1}{2\theta}\|(u^{k+1}+v) - I_0\|^2_{L^2(\Omega)} + \dfrac{\eta}{2}\|(u^{k+1}+v) - \dfrac{1}{2}\|^2_{L^2(\Omega)} \\ +\dfrac{\lambda_3}{2}\|e - \nabla(u^{k+1}+v) + g - c^k\|^2_{L^2(\Omega)} \\ +\dfrac{\lambda_4}{2}\|z - v - w^k\|^2_{L^2(\Omega)}\Big], \end{cases}$$

$$c^{k+1} = c^k - e^{k+1} + \nabla(u^{k+1} + v^{k+1}) - g,$$

$$w^{k+1} = w^k - z^{k+1} + v^{k+1}.$$

The constants λ_i, for $i = 1, 2, 3, 4$ are the fixed penalty parameters, used in the Bregman approach.

The minimization problem in subproblem C^k is solved by iteratively minimizing with respect u, d_1 and d_2, alternatively. Similarly, in subproblem T^k the minimization problem is also solved by iteratively minimizing with respect v, e and z, separately. Consequently we have that:

- The formula for u^{k+1} is derived from the Euler-Lagrange equation, *i.e.* it is the solution of the following PDE (partial differential equation) in Ω, hereafter denoted by *"cartoon PDE"*

$$\begin{aligned}\Big(\eta + \tfrac{1}{\theta} - (\lambda_1 + \lambda_2)\Delta\Big)u^{k+1} &= -\Big(\eta + \tfrac{1}{\theta} - \lambda_1\Delta\Big)v^k \\ -\lambda_1 \mathrm{div}(d_1^k + g - b_1^k) &- \lambda_2 \mathrm{div}(d_2^k - b_2^k) + \frac{1}{2}\eta + \frac{1}{\theta}u_0\end{aligned}$$

where Δ and div are the Laplace and divergence operators, respectively, with the non-homogeneous Neumann boundary condition on the boundary $\partial\Omega$ of Ω

$$\begin{aligned}\frac{\partial u^{k+1}}{\partial n} &= \frac{\lambda_1}{\lambda_1 + \lambda_2}(d_1 + g - b_1^k - \nabla v^k) \cdot n \\ &\quad + \frac{\lambda_2}{\lambda_1 + \lambda_2}(d_2 - b_2^k) \cdot n\end{aligned}$$

where n is the unit outward normal to $\partial\Omega$ and "$\cdot$" denotes the inner product in R^2.

- The formulas for d_1^{k+1} and d_2^{k+1} are explicit using the shrinkage operators, so at each pixel (i, j) in Ω

$$\begin{aligned}d_{1\,i,j}^{k+1} &= \mathrm{shrink}\big((\nabla(u^{k+1} + v^k) - g + b_1^k)_{i,j}, \tfrac{1}{\lambda_1}\big), \\ d_{2\,i,j}^{k+1} &= \mathrm{shrink}\big((\nabla u^{k+1} + b_2^k)_{i,j}, \tfrac{1}{\lambda_2}\big),\end{aligned}$$

where for z, γ in R

$$\mathrm{shrink}(z, \gamma) \;=\; \frac{z}{|z|} \cdot \max(|z| - \gamma, 0).$$

- The formula for v^{k+1} is derived again from the Euler-Lagrange equation, thus v^{k+1} is the solution of the following PDE in Ω, hereafter denoted by *"texture PDE"*

$$\begin{aligned}(\eta + \tfrac{1}{\theta} + \lambda_4 - \lambda_3\Delta)v^{k+1} \;&=\; -(\eta + \tfrac{1}{\theta} - \lambda_3\Delta)u^{k+1} \\ -\lambda_3 \mathrm{div}(e^k + g - c^k) + \lambda_4(z^k - w^k) &+ \tfrac{1}{2}\eta + \tfrac{1}{\theta}I_0,\end{aligned}$$

with the non-homogeneous Neumann boundary condition

$$\frac{\partial v^{k+1}}{\partial n} = (e - \nabla u^{k+1} + g - c^k) \cdot n, \quad \text{on } \partial\Omega.$$

- The formulas for e^{k+1} and z^{k+1} are also explicit using the shrinkage operators (likewise for d_1^{k+1} and d_2^{k+1}).

Summarizing, the split Bregman method for $\mathcal{P}$ is :

Algorithm

Input - Multisource images I_i, $i = 1, 2, \ldots, N$.

Initialize - $u^0 = I_0$, fix θ, η, α, λ_i $(i = 1, 2, 3, 4)$, tol,
and fix v^0, b_1^0, b_2^0, c^0, d_1^0, d_2^0, e^0 ω^0, z^0 equal to zero.

While $\max\{|u^k - u^{k-1}|, |v^k - v^{k-1}|\} > tol$

Cartoon part -

u^{k+1} solution of the *"cartoon PDE"*,

$d_1^{k+1} = \text{shrink}(\nabla(u^{k+1} + v^k) - g + b_1^k, \frac{1}{\lambda_1})$,

$d_2^{k+1} = \text{shrink}(\nabla u^{k+1} + b_2^k, \frac{1}{\lambda_2})$,

$b_1^{k+1} = b_1^k - d_1^{k+1} + \nabla(u^{k+1} + v^k) - g$,

$b_2^{k+1} = b_2^k - d_2^{k+1} + \nabla u^{k+1}$.

Texture part -

v^{k+1} solution of the *"texture PDE"*,

$e^{k+1} = \text{shrink}(\nabla(u^{k+1} + v^k) - g + c^k, \frac{1}{\lambda_3})$,

$z^{k+1} = \text{shrink}(v^{k+1} + w^k, \frac{\alpha}{\lambda_4})$,

$c^{k+1} = c^k - e^{k+1} + \nabla(u^{k+1} + v^{k+1}) - g$,

$w^{k+1} = w^k - z^{k+1} + v^{k+1}$.

End

Output - Fused image $u^k + v^k$, where u^k is the cartoon part and v^k the texture part.

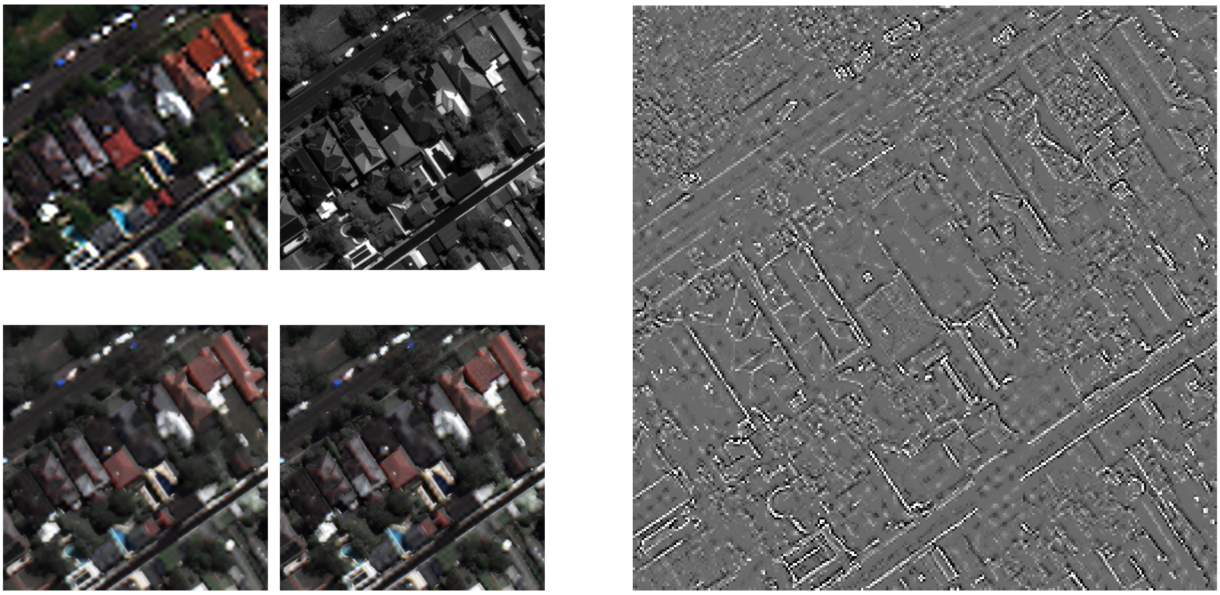

FIGURE 1. Top left: Multispectral (R,G,B) scene of Sidney, Australia. Top middle: Panchromatic image. Bottom left: Cartoon. Bottom middle: Fused Image. Right: Texture.

4. Experiments

In this section we apply the proposed technique to three different types of images. In the experiments, we use the following parameters $\eta = 10^2,\ \theta = 10^{-5},\ \lambda_1 = \lambda_3 = 1,\ \lambda_2 = 10, \lambda_4 = 10^4,\ \alpha = 1,$ and $tol = 10^{-4}$.

The first example, displayed in Figure 1, shows an EO (earth observation) image, obtained with the remote sensing satellite *WorldView-2*. The EO image was downloaded freely from Digital Globe Inc. (www.digitalglobe.com), and acquired on April 3rd 2011 over Sydney (Australia). For this experiment we consider the panchromatic band (with a high spatial resolution of 0.46 meters at nadir) and three visible multispectral bands (Red,Green,Blue) (with a low spatial resolution of 1.84 meters at nadir and a high spectral resolution). The goal is to fuse the high spatial and high spectral images in a single image and extract from it the cartoon and texture information. We fuse each multispectral band (64×64 pixels) with the panchromatic image (256×256 pixels), and to this end we first perform a pre-processing step, where each multispectral image is up-sampled to the panchromatic image size, by using a bi-cubic interpolation. In Figure 1 we see that our model produces a fused image with significant improvements. Also note that the cartoon is almost the same as the fused image, and the texture information is meaningful.

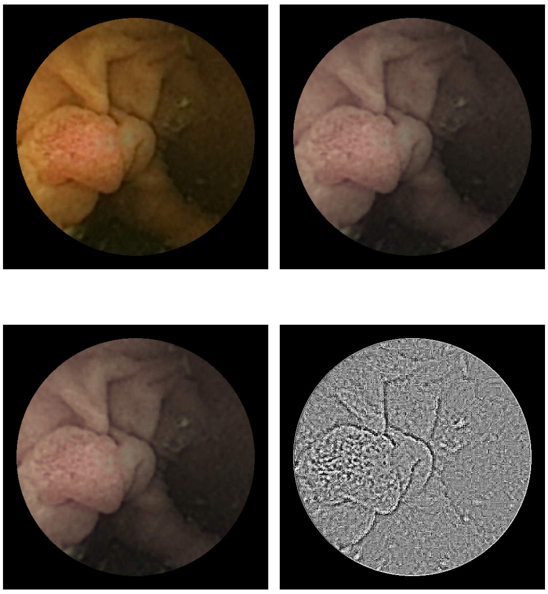

FIGURE 2. Top left: Polyp image obtained with Pillcam Colon 2, by courtesy of University Hospital of Coimbra, Portugal. Top right: Fused Image. Bottom left: Cartoon. Bottom right: Texture.

Figure 2 shows the results of our method applied to a medical (RGB) image (with 536×536 pixels), acquired with the wireless capsule Pillcam Colon 2 of *Given Imaging*. It displays a colonic polyp (the reddish region) exhibiting strong texture. Here we fuse each channel with the grayscale version of the three color image. Again the results confirm that the cartoon has almost all the details of the fused image, and the textural image captures the small features on the polyp surface.

Finally, Figure 3 shows the outcome of our method for a standard test image, downloaded from the IPOL archive (http://www.ipol.im/).

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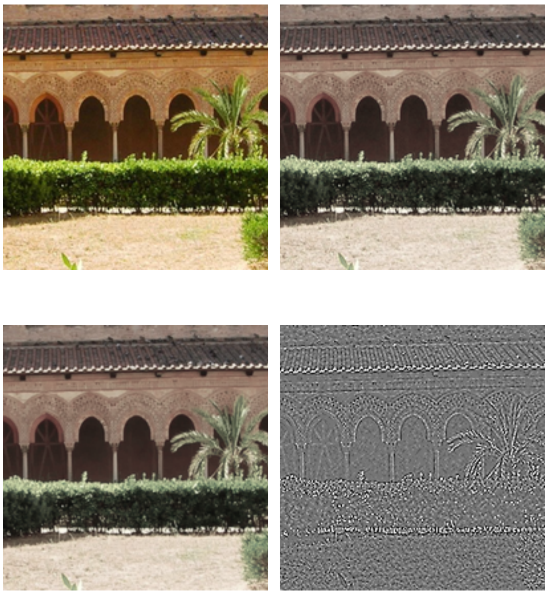

FIGURE 3. Top left: Garden standard test image. Top right: Fused Image. Bottom left: Cartoon. Bottom right: Texture.

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