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A ROBUST MODEL FOR OPTIMAL BANK ASSET STRUCTURE

CATARINA COELHO, JOSÉ LUIS SANTOS AND PEDRO JÚDICE

Abstract: Given a liability structure, the optimal bank asset structure problem consists in determining an asset allocation that maximizes profit, subject to restrictions on Basel III ratios and credit, liquidity and market risks. Most bank asset allocation models are very sensitive to inputs, making them difficult to use in practice, due to rigidities in the balance sheet. Our first contribution is to develop an optimization method that guarantees the stability of the allocations against the parameters, based on turnover constraints.

On the other hand, bank allocation models have not been tested using historical data. We develop such tests, which document the superior performance of optimization strategies when compared to heuristic rules, resulting in an average annual out-of-sample outperformance of 9.4% in terms of Return on Equity using our data set. The tests also confirm that turnover constraints are important in order to achieve smooth allocations that can be implemented in practice.

Keywords: bank asset structure, optimization, turnover constrains, out-of-sample performance.

1. Introduction

The optimization of bank balance sheets consists on the choice of the allocations in the different asset classes, and involves different variables, namely the prospective returns on asset classes, the regulatory framework, the accounting rules (IFRS) and also internal risk estimates.

The literature on bank balance sheet optimization has recently known significant advances. Most models, however, are sensitive to the input parameters, resulting in high changes in asset allocations. In the case of banks, this makes these asset allocations very difficult to implement in practice, as banks cannot grow on most of the asset classes by an arbitrary amount. For example, if a bank has an allocation of 20% of in mortgages, it is extremely

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difficult, if not impossible, to grow this allocation to 60%, due to supply factors and the operational effort involved. The first motivation of our research is to devise a robust framework for bank balance sheet optimization that yields smoother allocations.

To date, there have been no historical tests of bank balance sheet optimization methodologies, unlike for instance, in the subject of portfolio optimization, where numerous papers have addressed the historical performance of different optimization methodologies. As many papers have shown in the case of portfolios (see for instance [5]), optimization methods may guarantee the best returns ex-ante (or in-sample), but in many cases do not outperform heuristic strategies ex-post (out-of-sample). Our second goal of the paper is to devise a testing framework that allows us to evaluate the stability of allocations and the out-of-sample performance of different optimization and heuristic strategies.

Bank balance sheet optimization models have been available since the eighties. We cite a few references. Kusy and Ziemba [12] have created a framework for calculating optimal balance sheets using the stochastic nature of cash outflows. Kosmidou and Zopounidis [11] have devised a simulation-optimization framework taking into account the interest rate risk in the balance sheet. The regulatory and accounting framework have evolved considerably since then: for instance, Basel III has been implemented, and regulators actively monitor capital and liquidity ratios; credit risk measurement has also evolved considerably since then.

A few papers have recently addressed the optimization of balance sheets. Halaj [9] has devised a methodology for calculating the optimal asset structure of a bank in the presence of solvency and liquidity restrictions. The method is quite broad, in that it includes both interest rate and credit risk, and also regulatory compliance. However, it turns out to be sensitive to the input parameters and the model has not yet been tested using historical data to assess the stability of allocations and the out-of-sample performance of the model versus other heuristic allocation models.

Schmaltz *et al.* [24] have also undertaken innovative research on optimal balance sheets in the presence of regulatory constraints, which demonstrates the power of optimization techniques in solving concrete financial problems and their superiority to commonly accepted heuristic techniques. The goal of this model is different from our research, in that the departing point is a Basel III non-compliant bank and the methodology determines the least costly way to comply with the Basel III ratios, whereas the goal of our research is to dynamically determine the optimal allocations even assuming compliance with Basel III.

The authors do not address the differentiation between legacy loans and new loans, that is present for instance in [4] and [9], which is important to determine the prospective net interest margins on different asset classes. Let us give an example: suppose that rates have been falling, legacy loans have an interest rate of 5%, and new loans have an interest rate of 2%. Measuring the prospective return at 5% becomes too optimistic, whereas using 2% becomes very pessimistic. The rate on this asset class will be somewhere in between 2% and 5%, depending on the turnover of the assets.

The model also assumes no lower bounds on the deleveraging of asset classes, which in practice may not happen or may prove difficult. For example, unlimited deleveraging in long-term mortgages may prove difficult, particulary for many geographies outside the United States, where securitization markets are much less active. Also, the model assumes that deleveraging occurs with a linear penalty, which may be hard to estimate. Let us give another example: suppose a bank wants to decrease its mortgages by 20% in a year and the turnover on this asset class is of 5%. The first 5% would be relatively easy to implement, in that no new mortgages would be given, with a zero penalty. It would be hard to estimate the impact of the remaining 15%, as there would be several possible combinations: one can sell business lines or subsidiaries, or simply sell a portfolio of credits. However, implementation costs are difficult to estimate. The haircut on selling credit business lines or credit portfolios depends largely on the credit risk of those portfolios. The impact of job cuts is hard to estimate, as it depends on the number of people in the business lines and their years of service; also, research shows that job cuts have many indirect costs which are hard to quantify, namely the costs of litigation and of decreasing quality of services (see for instance [23] or [26]). This example shows the non-linear nature of the deleveraging and also the difficulties in calibrating this penalty in practice. The authors circumvent this difficulty by estimating a shadow price assuming that the bank is Basel II - optimal. We believe that this methodology is difficult to use in practice, as many banks have not behaved optimally in the past, resulting in excessive risk and low profitability in many cases as financial crises have shown.

Finally, it would be useful to test their model against historical data to assess the out-of-sample performance of the model. As we have said before, there is no ex-post guarantee that an optimization framework will yield superior results when compared to heuristic strategies.

With this literature review in mind, our research provides the following contributions:

(1) First, we devise an optimal bank asset allocation model, given a liability structure, using global turnover constraints, which are easy to calibrate, and have been used in the context of portfolio optimization [6]. Turnover constraints prevent large fluctuations in allocations each year. Consequently, a major change in the balance sheet is only achieved if market conditions show a steady trend over time. We focus solely on the asset structure, given that it is easier to change the asset structure than the liability structure. For example, as documented in [8], growing the deposit base is often difficult as retail deposits tend to be sticky. Also, equity capital may be difficult to obtain particularly at times of increased financial stress.

(2) We use extensive historical data to devise a testing framework of optimization and heuristic strategies, addressing both the performance and the stability of the allocations.

(3) We document the excessive sensitivity of optimization methodologies without global turnover constraints. The allocations without turnover constraints can vary in our setting up to 40% in a year, which is infeasible in practice. For example, a bank cannot change the allocation of its consumer credit portfolio from 20% to 60% in a year, unless it makes an acquisition of a large consumer credit business unit, which may not be readily available. We demonstrate that turnover methodologies yield smoother allocation trajectories, which enable them to be used in practice.

(4) Finally, we document the superiority of optimization strategies when compared to classical heuristic strategies, resulting in an average out-of-sample outperformance of 0.94% in return on assets and of 9.4% in return on equity.

These contributions yield a model with turnover restrictions, which is suitable to be used in practice, since it combines the superior profitability presented by the optimization models with allocations that can be implemented by bank management.

This paper is organized as follows: section 2 describes the model and the return and risk parameters used as inputs; section 3 conducts an extensive analysis of out-of-sample results on the performance and stability of the optimization method with turnover constraints against a series of heuristic benchmarks that we adapt to the banking context. Section 4 concludes the manuscript.

2. Model description

In this section, we develop the model for the optimal asset structure. We assume that the liability structure is fixed, for the reasons explained in the introduction.

We intend to apply the algorithm regularly over the period under study. Consequently, the algorithm starts with the portfolio x_0, which is the one currently used in the first year of the study. Next, it finds the optimal asset structure x^*, which satisfies the Basel III and turnover constraints, given a prediction of the rates, defaults and risk of each asset. Then, x^0 is updated with x^* and the model is applied to obtain the optimal solution for the following year and so on.

2.1. The proposed model. Let $A = \{A_1, A_2, A_3, A_4, A_5 A_6, A_7\}$ be the set of assets described in the following table:

Asset	Description
A_1	Cash
A_2, A_3	Mortgage and Personal Loans, respectively
A_4, A_5	Treasury bonds AFS and HTM, respectively
A_6, A_7	Corporate bonds AFS and HTM, respectively

This set describes a great part of the activity of many banks, and for all these aggregates we have historical data as we will see in the following sections. HTM designates *Held-To-Maturity* assets, while AFS designates *Available-For-Sale* assets. Let $A_L = \{A_2, A_3, A_5, A_7\}$ be the subset of assets associated with long holding periods, that is, loans and HTM assets. Additionally, define $\Omega = \{x \in \mathbb{R}^{\#A} : \sum_{i \in A} x_i = 1, x_i \geq 0\}$ as the set of admissible portfolios. Our model distinguishes between legacy ($\hat{x}_i$) and new ($\tilde{x}_i$) contracts for each asset. As consequence, we also distinguish between the interest rate on legacy contracts, $\hat{r}_i$, and the interest rate on new ones, r_i. The amount of legacy contracts is obtained through repayments, so that

$\hat{x}_i = (1-\alpha_i)x_i^0$, where α_i is the repayment portion on asset i. Finally, legacy and new contracts fulfill the portfolio, so $x_i = \hat{x}_i + \tilde{x}_i$, $\forall i \in A$.

Since capital and liabilities are given as inputs to the problem, the calculations that depend on them are considered constants. For example, the numerator in the common equity tier I ratio is a constant in the problem. Given the set of inputs, we propose the following model:

$$
\begin{aligned}
\max_{x\in\Omega} \quad & r(x) \\
\text{subject to} \quad & \frac{\sum_{i\in A}\lambda_i x_i}{\Lambda} \geq K_1, && (1)\\
& \frac{N}{\sum_{i\in A}\nu_i x_i} \geq K_2, && (2)\\
& \frac{\sum_{i\in A} S_i x_i}{M} \geq K_4, && (3)\\
& \frac{C - IRR - \sqrt{V(x)}}{\sum_{i\in A} RW_i x_i} \geq K_3, && (4)\\
& \hat{x}_i = (1-\alpha_i)x_i^0, \qquad \forall i \in A && (5)\\
& x_i = \hat{x}_i + \tilde{x}_i, \qquad \forall i \in A && (6)\\
& x_i - x_i^0 \leq y_i, \qquad x_i^0 - x_i \leq z_i, \qquad \forall i \in A && (7)\\
& \sum_{i\in A}(y_i + z_i) \leq h && (8)\\
& y_i \leq \alpha_i x_i^0, \qquad \forall i \in A_L && (9)\\
& z_i \leq \alpha_i x_i^0, \qquad \forall i \in A_L && (10)\\
& y_i,\, z_i \geq 0, \qquad \forall i \in A && (11)
\end{aligned}
$$

The intention of this problem is to maximize the return $r(x)$, constrained to (1)-(4) corresponding, respectively, to the risk and regulatory restrictions posed by Basel III: the Liquidity Coverage Ratio (LCR), which compares liquid assets with net cash outflows in 30 days, that is, in the short term; the Net Stable Funding Ratio (NSFR), which compares the medium-term liquidity of assets with medium-term financing stability; a liquidity stress coverage ratio that compares liquid assets with wholesale liabilities, which determines if the bank is well prepared for liquidity dry ups; and Common

Equity Tier 1 (CET1) after a solvency shock $V(x)$ and an interest rate shock IRR, which we explain below. This set of restrictions was taken from [10], and are important to ensure the sustainability of the balance sheet.

Constraints (5) and (6) allows us to distinguish between legacy and new assets. The main novelty of the model is the inclusion of turnover constraints, present in the context of portfolio optimization, but not included so far in the bank balance sheet optimization literature. The turnover constraints (7)-(11) allow us to achieve some stability over the years and to use the solutions in real situations. The local turnover constraints (9)-(10) are applied only to assets in A_L, since the banks can only reinvest, to a great extent, the amount that comes from repayments. Although the theme of limited reinvestments by repayments has also been addressed in [9], that research neither considers the upper bound local turnover constraint (9) nor the global turnover constraint (8). Consequently, that model allows an arbitrarily large investment in assets belonging to A_L which can not be easily divested in the following years. Additionally, the global turnover is not restricted, leading to solutions that show large variations which are hard to accomplish in practice.

2.2. Model parameters estimation. As we already mentioned, we assume that the liability structure is fixed and corresponds to:

	Liabilities			
	Deposits	Money Market	Issued Bonds	Capital
Allocation	0.5	0.2	0.2	0.1

$\Lambda = 21.5\%$	$\lambda = [100\%\ 0\%\ 0\%\ 100\%\ 100\%\ 50\%\ 50\%]$
$N = 78\%$	$\nu = [0\%\ 65\%\ 85\%\ 5\%\ 5\%\ 5\%\ 5\%]$
$C = 10\%$	$IRR = 1.1\%$
$M = 40\%$	$\Delta = [0\%\ 0\%\ 0\%\ 0\%\ 0\%\ 0\%\ 0\%]$
$h = 15\%$	RW=$[0\%\ 35\%\ 100\%\ 0\%\ 0\%\ 100\%\ 100\%]$
$K_1 = K_2 = 110\%$	$S = [100\%\ 0\%\ 0\%\ 100\%\ 100\%\ 100\%\ 100\%]$
$K_3 = 10\%$	$K_4 = 100\%$

TABLE 1. Fixed input data for the model

Table 1 reports the fixed input data to our model. In this table, IRR represents the sensitivity of net interest income to a 300 basis point shock in interest rates. Since all the asset classes are at fixed rates, the liability

A_i	A_1	A_2	A_3	A_4	A_5	A_6	A_7
M_i	—	30	2	—	10	—	20
α_i	1	1/30	1/2	1	1/10	1	1/20
LGD_i	0	0.471	0.64	0	0	0.628	0.628

TABLE 2. Values for the fixed parameters used in the model for each asset.

structure determines completely this sensitivity. In our test case, a 300 basis point increase in rates has a negative impact of 1.1% in the balance sheet.

In this model, we set $\alpha_i = 1/M_i$, $\forall i \in A_L$, where M_i is the maturity (in years) and $\alpha_i = 1$ in the remaining cases (see Table 2) meaning that there is no legacy when $i \notin A_L$. Additionally, the interest rate on legacy contracts of asset i, $\hat{r}_i$, is initialized as the average rate of the previous 10 years for the first year in our study. Subsequently, $\hat{r}_i$ is updated with $(1-\alpha_i)\hat{r}_i + \alpha_i r_i$, $\forall i \in A$.

The objective function of our model corresponds to the prospective return on legacy and new loans and is given by:

$$r(x) = \sum_{i \in A_L} (\hat{x}_i \hat{r}_i + \tilde{x}_i r_i - x_i LGD_i PD_i) + \sum_{i \notin A_L} x_i r_i,$$

where r_i and PD_i are the estimated interest rate and the estimated probability of default on asset i. These value were estimated using the (simple) moving average method over the previous 10 years (see [20, 22]). The LGD_i parameters (loss given default) are reported in Table 2 and were obtained from [27, 3] for loans and from [25] for corporate bonds. We assumed that $LGD_i = 0$, $\forall i \notin A_L$.

We would like to emphasize that the prospective return r_i on AFS bonds is given by j_i, where j_i is the yield on AFS bonds at the beginning of the year under study.

The Common Equity Tier 1 constraint (4) depends on $V(x) = \sum_{i \in A} (\sigma_i x_i)^2$ where σ_i is a risk penalty parameter associated to asset i. We assume that $\sigma_1 = \sigma_5 = 0$ since their credit risk is very low. $V(x)$ is a penalty function for the asset structure that depends on the individual risk penalties for each asset class.

The penalty risk for loans and corporate bond HTM will be given by the credit VaR which is given by the difference between the unexpected loss at 99.9% and the expected loss [7], that is $\sigma_i = UL_i(0.999) - EL_i$ where

Parameter	Value
ρ_2	0.15
ρ_3	$0.03 \times \frac{1-\mathrm{e}^{-35\overline{PD}_3}}{1-\mathrm{e}^{-35}} + 0.16 \times \left(1 - \frac{1-\mathrm{e}^{-35\overline{PD}_3}}{1-\mathrm{e}^{-35}}\right)$
ρ_7	$0.12 \times \frac{1-\mathrm{e}^{-50\overline{PD}_7}}{1-\mathrm{e}^{-50}} + 0.24 \times \left(1 - \frac{1-\mathrm{e}^{-50\overline{PD}_5}}{1-\mathrm{e}^{-50}}\right)$

TABLE 3. Value of parameter ρ_i, $i \in \{2, 3, 7\}$.

$$UL_i(0.999) = N\left(\sqrt{\frac{1}{1-\rho_i}} \times N^{-1}\left(\overline{PD}_i\right) + \sqrt{\frac{\rho_i}{1-\rho_i}} \times N^{-1}(0.999)\right) \times LGD_i$$

and $EL_i = \overline{PD}_i \times LGD_i, i \in \{2, 3, 7\}$. In these formulas, $\overline{PD}_i$ represents the average of PD_i over the previous 10 years and ρ_i corresponds to the correlation between different contracts of the same asset. Table 3 report the ρ_i values, $i \in \{2, 3, 7\}$, taken from [1, 21, 2].

Finally, the risk penalty for AFS bonds will be given by the Market VaR which is

$$N^{-1}(0.95)s_i, i \in \{4, 6\},$$

where s_i is a prediction of the standard deviation of the return of asset i from the previous 10 years.

3. Historical data description

We use historical data used to evaluate the performance of the proposed model, namely public USA data for interest rates and defaults from 1985 until 2016. Table 4 indicates the data sources, and Table 5 reports a summary overview of the average return and the average risk penalty on each asset.

4. Computational experiments

In this section, we report an extensive computational study comparing the proposed model with classical heuristic strategies, and we evaluate the ex-post performance, using historical data, in section 3.

Since the heuristic approaches may not give a solution that is compliant under Basel III, we search the nearest solution that verifies these constraints by solving the following model:

Asset structure	**Interest rate**	**Defaults**
Cash	[17]	—
Mortgage loans	[14]	[16, 27]
Personal Loans	[18]	[15, 3]
Treasury bonds AFS	[13]	—
Treasury bonds HTM	[13]	—
Corporate bonds AFS	[19]	[25, Exhibit 23 (page 28) and Exhibit 7 (page 8)]
Corporate bonds HTM	[19]	[25, Exhibit 23 (page 28) and Exhibit 7 (page 8)]

TABLE 4. Sources for the rates for each asset.

Asset structure	**Return** (%)	**Risk penalty** (%)
Cash	2.7917	0
Mortgage loans	5.6116	4.2690
Personal Loans	9.5912	7.3581
Treasury bonds AFS	5.8829	8.6807
Treasury bonds HTM	4.4000	0
Corporate bonds AFS	7.8829	7.3946
Corporate bonds HTM	6.8010	1.3915

TABLE 5. Average return and risk penalty for each asset during the evaluation period $(1985-2016)$.

$$\begin{array}{rl} \min\limits_{x\in\Omega} & \|x - x^H\| \\ \text{subject to} & (1)-(11), \end{array}$$

where x^H is a heuristic solution. In this research, we consider norm ℓ_1, that is, $\|x-x^H\| = \sum_{i\in A} |x_i - x_i^H|$, since it allows us to get solutions that modify fewer components of the original one, making this approach less sensitive to parameters than other norms, such as the Euclidean norm. However, other norms could be considered.

4.1. Tested approaches. Altogether, six approaches (three optimized and three heuristics) were tested in this work. Three of them come from the optimized model suppressing some turnover constraints to better understand their effect in the final solution. These strategies are:

- M1: this approach consists of applying the original model presented in section 2;
- M2: similar to the previous one but removing upper bound local turnover constraint (9);
- M3: similar to the previous one but removing also the global turnover constraint (8);

We compare the optimized approaches against classical heuristic approaches, which we list below:

- EW (Equal Weighting): all the assets have the same allocation in the balance sheet;
$$x^{EW} = \Big[\frac{100}{7}\% \; \frac{100}{7}\% \; \frac{100}{7}\% \; \frac{100}{7}\% \; \frac{100}{7}\% \; \frac{100}{7}\% \; \frac{100}{7}\%\Big].$$
- 60/40: this is an adaptation of the 60/40 equity/bond portfolio allocation [5] to banks. In this work, we use this strategy to define a balance sheet allocating 60% in assets with high risk and 40% to assets with lower risk. We consider the cut-off point for risk as 2% and set equal weightings inside each group. Taking into account the average risk reported in Table 5, this leads to the following allocation:
$$x^{60/40} = \Big[\frac{40}{3}\% \; \frac{60}{4}\% \; \frac{60}{4}\% \; \frac{60}{4}\% \; \frac{40}{3}\% \; \frac{60}{4}\% \; \frac{40}{3}\%\Big].$$
- RP (Risk Parity): this strategy makes the allocations in such a way that all the assets contribute with the same risk to the final solution. Then, in case $\sigma_i > 0$, $\forall i \in A$, this solution could be defined as $x_i = \frac{1/\sigma_i}{\sum_{j\in A} 1/\sigma_j}$, $i \in A$. However, some assets have no risk or very low risk, so that we need to adapt the methodology to our bank setting. Consequently, for a specific year of the simulation, we define the set of assets A_R that have a risk penalty greater than 2% and apply a risk parity strategy to these assets, and an equal weighting strategy to the remaining ones. This set has to be updated for each year in the simulation. To compare the behaviour of this solution with the previous one, we keep the same allocation proportion between high-risk and low-risk assets. Thus, this solution is given by
$$x_i^{RP} = 0.6 \times \frac{1/\sigma_i}{\sum_{j\in A_R} 1/\sigma_j}, \quad i \in A_R.$$

	Initial balance sheet						
Assets	**A**	**B**	**C**	**D**	**E**	**F**	**G**
Cash	50%	10%	$\frac{100}{7}$%	5%	20%	25%	10%
Mortgage loans	10%	20%	$\frac{100}{7}$%	40%	0%	0%	60%
Personal Loans	10%	30%	$\frac{100}{7}$%	20%	0%	50%	0%
Treasury bonds AFS	7.5%	15%	$\frac{100}{7}$%	25%	40%	0%	10%
Treasury bonds HTM	7.5%	10%	$\frac{100}{7}$%	5%	20%	25%	10%
Corporate bonds AFS	7.5%	10%	$\frac{100}{7}$%	2.5%	20%	0%	5%
Corporate bonds HTM	7.5%	5%	$\frac{100}{7}$%	2.5%	0%	0%	5%

TABLE 6. Description of the seven initial balance sheets (corresponding to the balance sheet for 1994 in our simulation).

and

$$x_i^{RP} = \frac{0.4}{\#(A \backslash A_R)}, \quad i \notin A_R.$$

4.2. Initial balance sheet. In order to assess the robustness of the results on the performance of the optimized and heuristic strategies, we defined seven different initial balance sheets (see Table 6), which are defined as follows:

- **A::** allocate 50% of the balance sheet to *Cash*;
- **B::** allocate 50% of the balance sheet to *Loans*;
- **C::** distribute the asset allocation evenly;
- **D::** typical asset structure of a diversified *retail bank*;
- **E::** typical asset structure of an *investment bank*;
- **F::** typical asset structure of a *consumer credit bank*;
- **G::** typical asset structure of a *mortgage loan bank.*

All of these initial balance sheets are compliant under Basel III.

4.3. Out-of-sample results. Although we have available data from 1985 to 2016, in order to obtain a solution in the *ex-ante* optimization process, we need to predict some parameters of our model (PD_i, r_i, σ_i), smoothing their values with the average over the previous 10 years, as discussed in section 2.2. Consequently, our *ex-post* simulation only runs from 1995 to 2016.

	Initial balance sheet						
Strategy	A	B	C	D	E	F	G
M_1	470.36	526.55	527.74	531.42	416.82	497.37	518.95
M_2	487.60	537.43	535.98	560.24	488.55	520.11	549.71
M_3	540.90	549.29	545.82	572.83	532.42	534.36	577.24
EW	424.24	456.07	453.99	461.46	349.37	449.42	445.21
60/40	431.15	462.40	459.67	466.38	349.37	449.36	447.88
RP	409.57	454.90	452.75	455.48	351.99	436.09	441.25

TABLE 7. Accumulated effective return in the last year of the simulation.

The *ex-post* simulation consists on evaluating the solution obtained in the *ex-ante* optimization process with the effective return function:

$$\tilde{r}(x) = \sum_{i \in A_L} (\hat{x}_i \hat{r}_i + \tilde{x}_i r_i - x_i LGD_i PD_i) + \sum_{i \notin A_L} x_i r_i,$$

where r_i is the actual value of the interest rate in the beginning of the year under study and PD_i is the default rate observed at the end of that year. For $i \notin A_L$, the effective return r_i is given by

$$r_i = j_i + \frac{dP(j)}{dj} \Delta,$$

where j_i is the yield on AFS bonds in the beginning of the year, P is the bonds price and Δ is the increase/decrease in interest rates from the beginning to the end of the year.

Figure 1 shows the evolution of the accumulated effective return, r^a, for each one of the strategies tested and the initial balance sheets considered. It is computed as

$$r_0^a = 100 \text{ and } r_t^a = r_{t-1}^a (1 + \tilde{r}(x(t))), t > 0,$$

where t is the index of each one of the years under study and $x(t)$ is the corresponding solution for that year.

The evolution of the allocation for these solutions is presented in Figures 2 - 8 (see the annex section). Table 7 summarizes the accumulated effective return in the last year of the *ex-post* simulation and Table 8 compares the average results of optimized approaches with the heuristic ones.

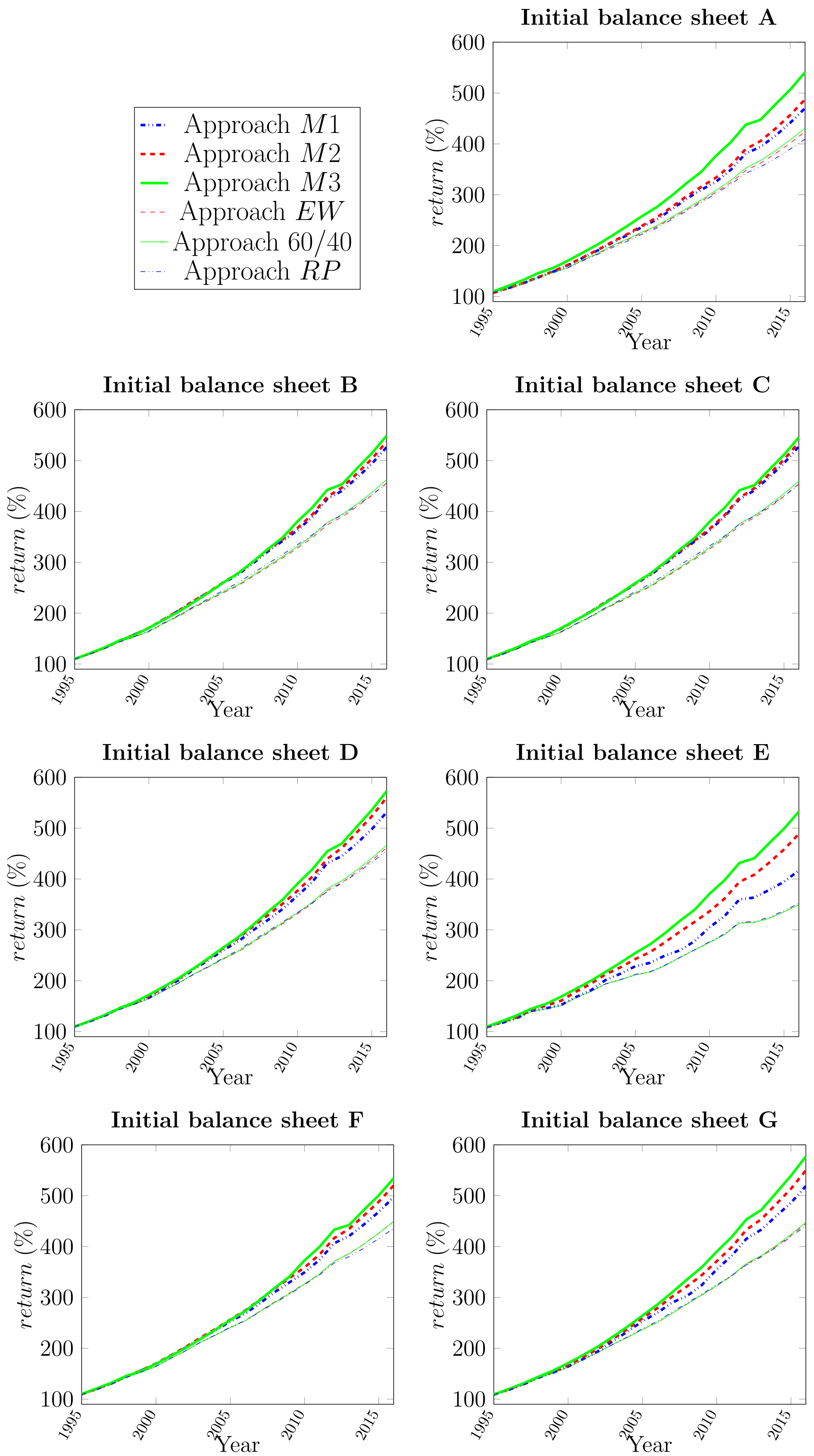

FIGURE 1. Accumulated effective return from 1995 to 2016.

	Initial balance sheet							
Strategy	A	B	C	D	E	F	G	Average
Optimized	7.586	7.946	7.935	8.100	7.383	7.756	8.045	7.822
Heuristic	6.760	7.159	7.135	7.195	5.863	7.021	7.019	6.879
Difference	0.827	0.787	0.800	0.905	1.520	0.735	1.026	0.943

TABLE 8. Comparison between the average accumulated effective return on both types of strategy (values in percentage).

These results attested the greater performance of the optimized solutions over heuristics in the ex-post simulation, reaching an average annual outperformance of 0.94% in terms of return on assets and 9.4% in terms of return on equity, since we consider a capital allocation of 10%.

When the constraints are removed from the model presented in section 2, the performance increases but the stability is penalized. For example, the strategy $M3$, which does not use turnover constraints, shows annual variations in the allocations above 40%, which does not happen in practice, as the allocations in the banking sector tend to be rigid for the reasons explained in the introduction.

The heuristic approaches show similar out-of-sample performances at the end of the simulation. Finally, as the initial balance sheets are different from the heuristic solutions, we can observe that the allocations converge to the to the heuristic solution.

5. Conclusion

The research we have undertaken was motivated by a practical problem in a banking context. When trying to implement a bank balance sheet optimization model, the allocations showed an excessive sensitivity to parameters, and could not be implemented in practice.

We tackled this problem using turnover constraints, present in the portfolio optimization literature. Using these types of constraints, we developed an optimization method, which is simple to implement in practice. In the paper, we also describe the estimation for the parameters.

Our second contribution was to develop a testing framework for different bank allocation strategies, based on an extensive historical data set, in order to evaluate the out-of-sample performance and stability of each strategy.

Our testing framework allowed us to confirm a series of conclusions:

(1) When we remove global turnover constraints from the optimization models, this results in excessive variations in the allocations that cannot be implemented in practice.
(2) Optimization models with turnover constraints show smoother allocations and therefore can be implemented in an industrial context.
(3) Optimization models show superior out-of-sample performance when compared to heuristic strategies. Notice that this is not a given, as numerous studies have shown that, in the case of portfolio optimization, often heuristic strategies outperform portfolio optimization strategies when considered out-of-sample. Using our dataset, we report an increase in annual ex-post outperformance of 0.94% in terms return on assets and of 9.4% in terms return on equity.

We hope that this research can contribute to the development of balance sheet optimization tools that can be used in practice by the banking sector.

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Catarina Coelho
Department of Mathematics, University of Coimbra, Apartado 3008, 3001–501 Coimbra, Portugal

E-mail address: catarinacoelho1995@gmail.com

José Luis Santos
CMUC, Department of Mathematics, University of Coimbra, Apartado 3008, 3001–501 Coimbra, Portugal

E-mail address: zeluis@mat.uc.pt

Pedro Júdice
Montepio Bank and ISCTE Business Research Unit, Portugal

E-mail address: PAJudice@montepio.pt

Annex

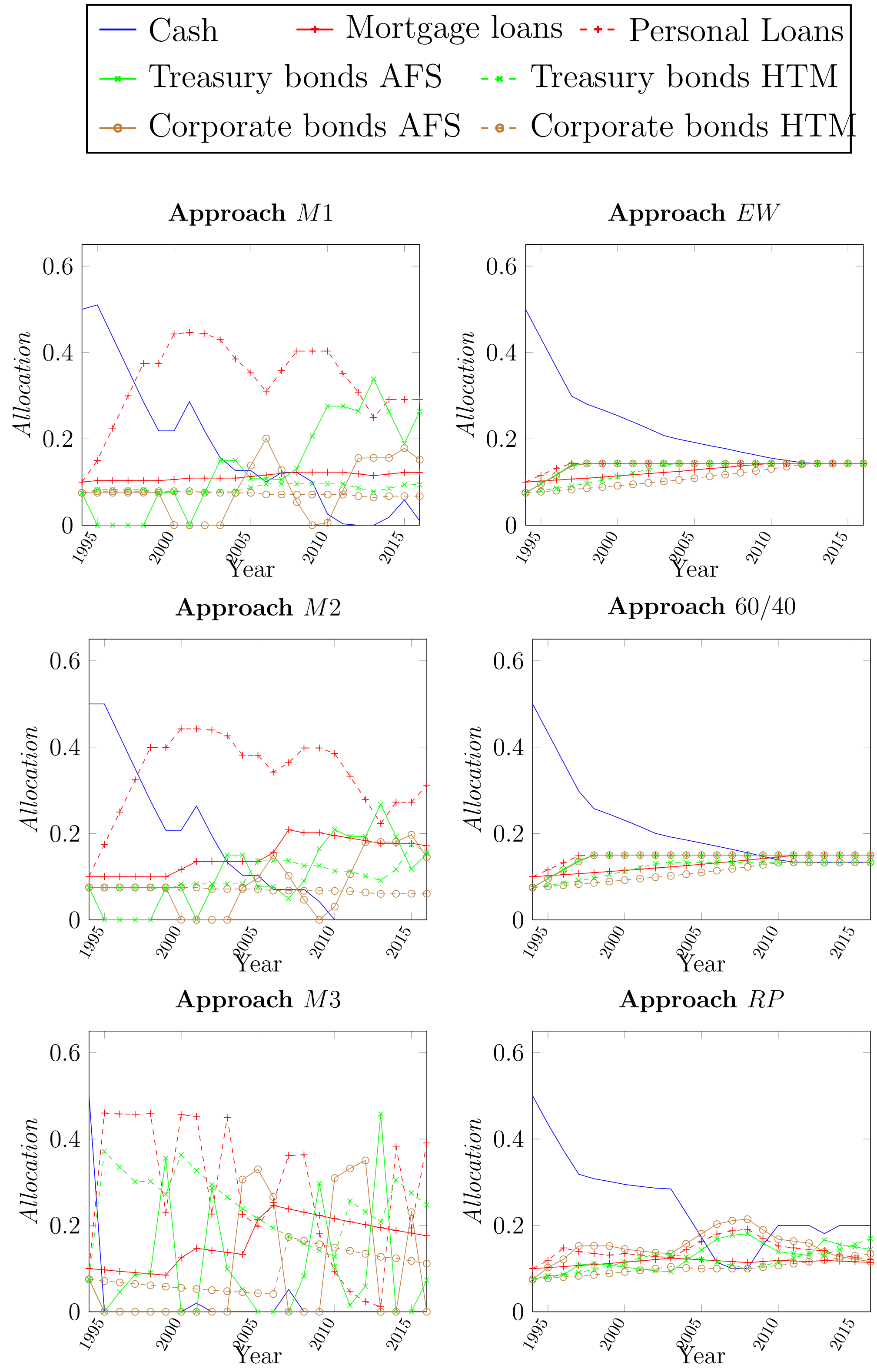

FIGURE 2. Evolution of the balance sheet during the simulation of the tested approaches over the period 1994 - 2016 (initial balance sheet A).

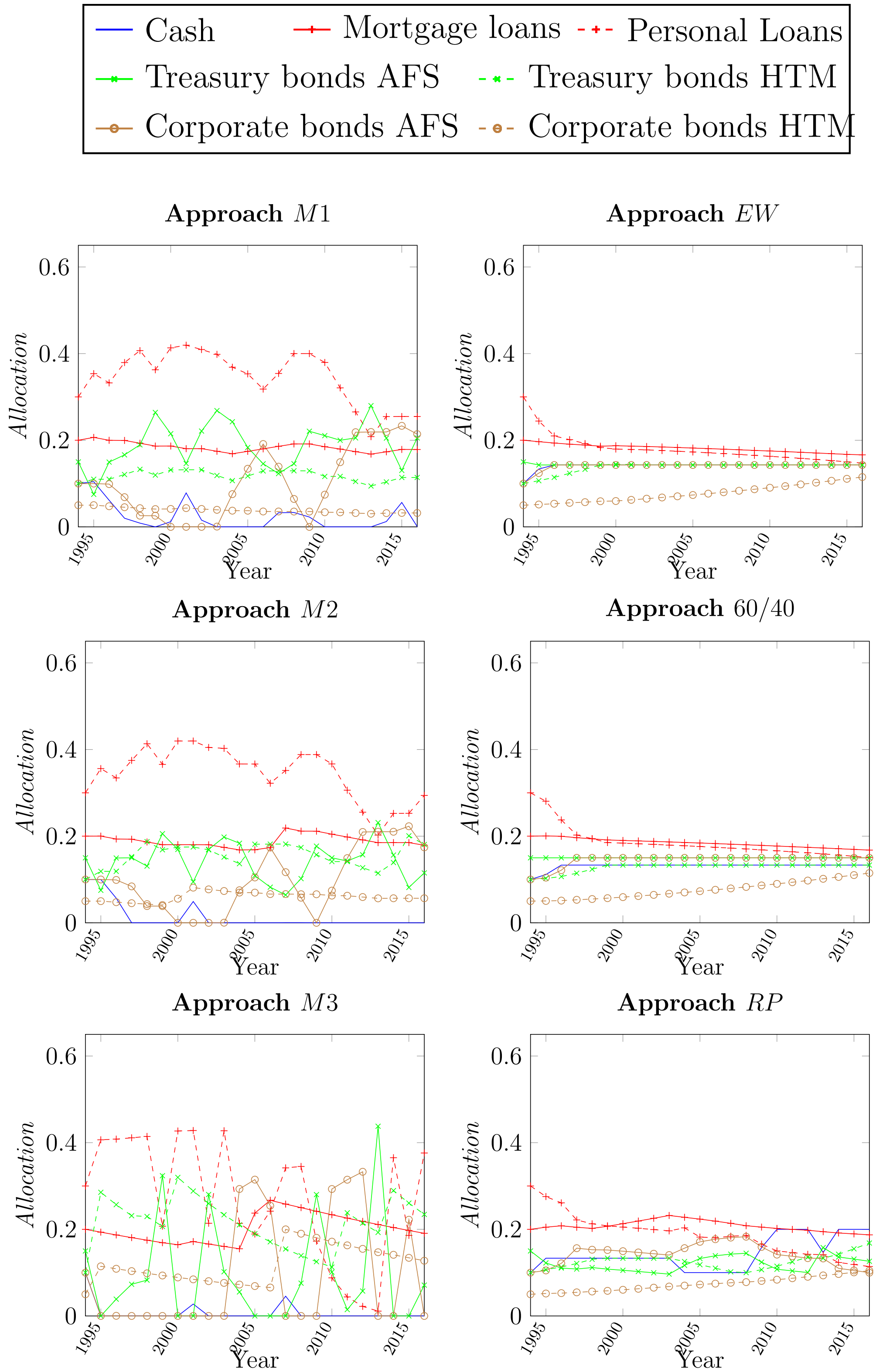

FIGURE 3. Evolution of the balance sheet during the simulation of the tested approaches over the period 1994 - 2016 (initial balance sheet B).

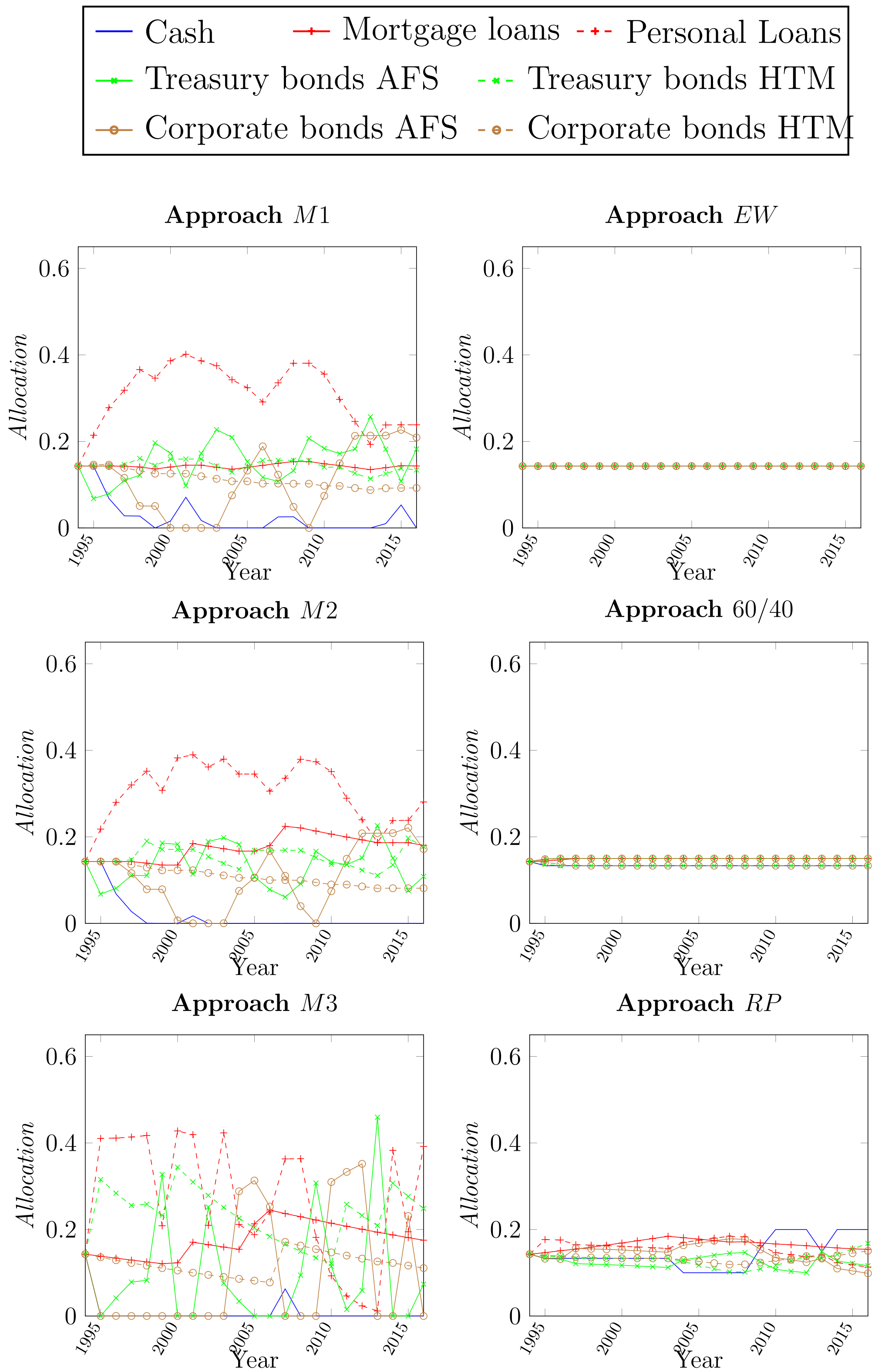

FIGURE 4. Evolution of the balance sheet during the simulation of the tested approaches over the period 1994 - 2016 (initial balance sheet C).

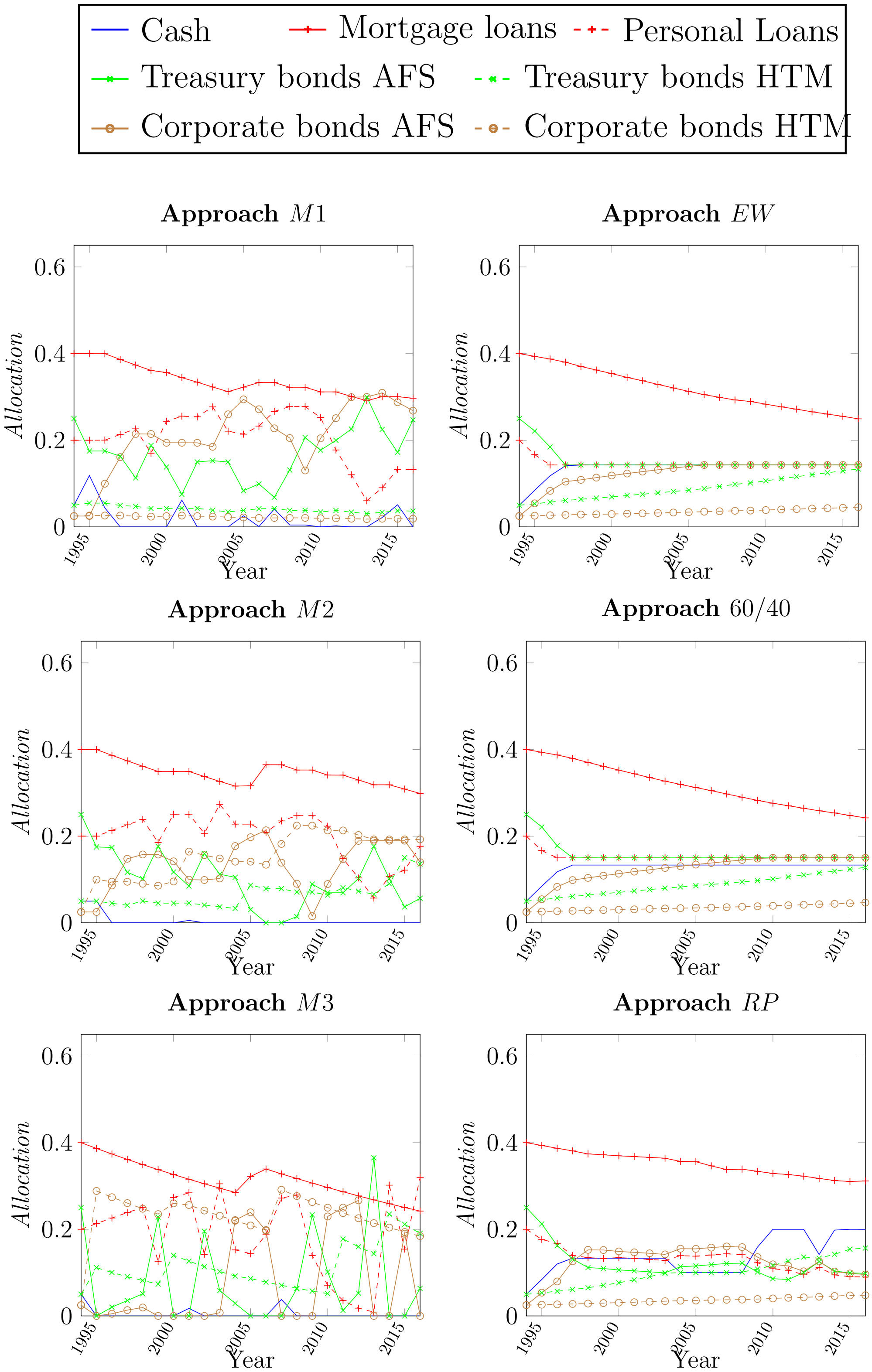

FIGURE 5. Evolution of the balance sheet during the simulation of the tested approaches over the period 1994 - 2016 (initial balance sheet D).

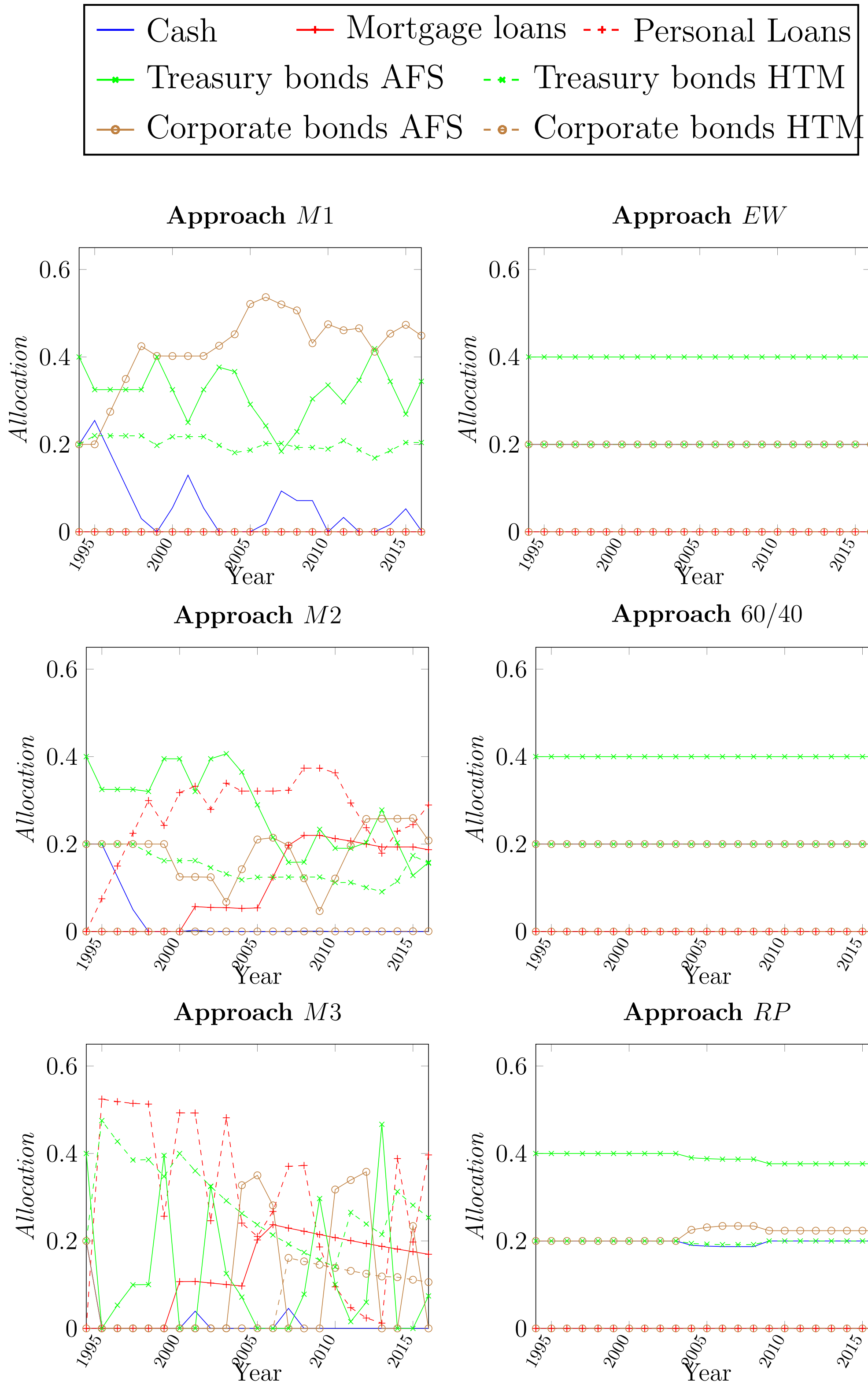

FIGURE 6. Evolution of the balance sheet during the simulation of the tested approaches over the period 1994 - 2016 (initial balance sheet E).

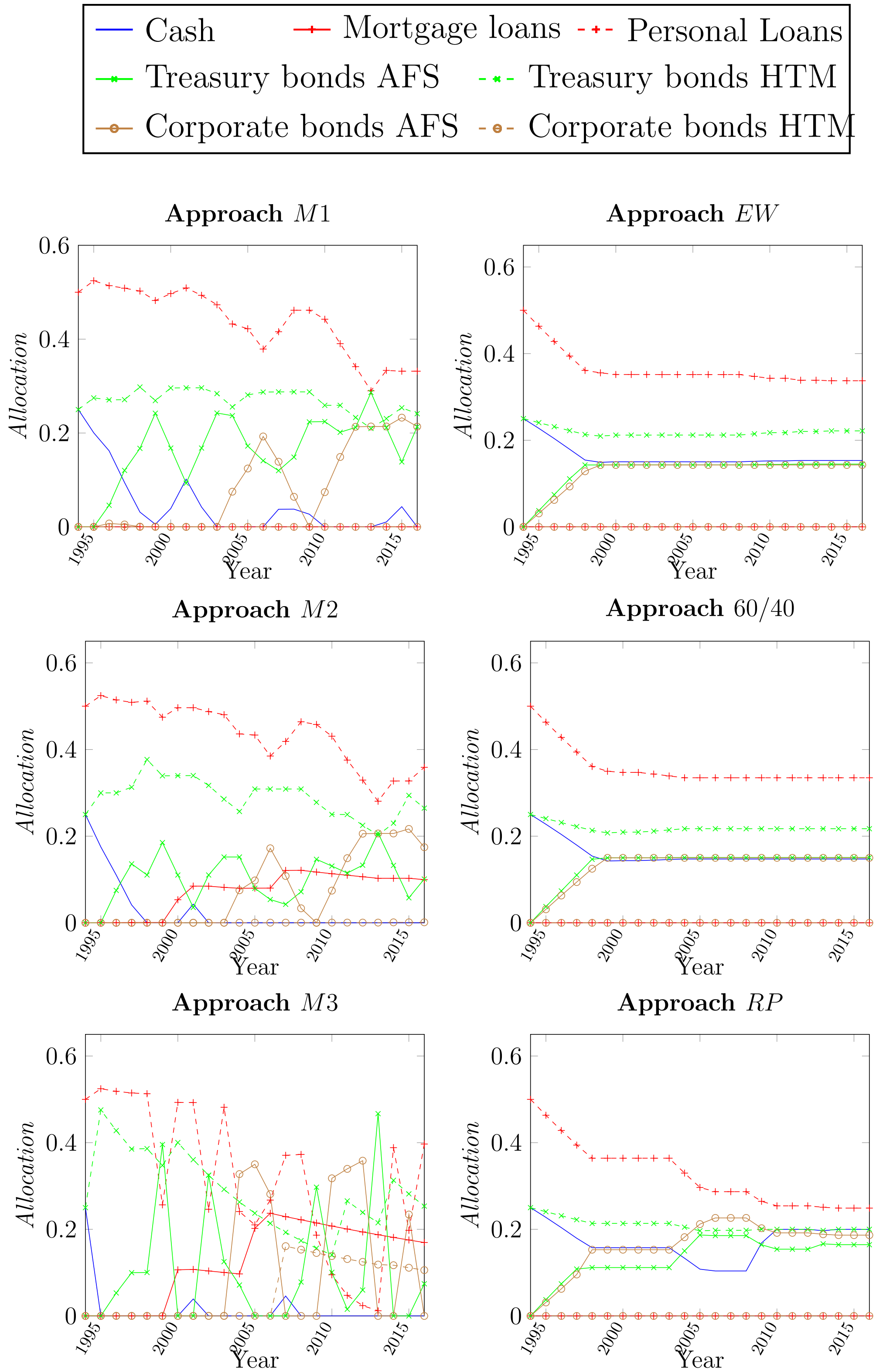

FIGURE 7. Evolution of the balance sheet during the simulation of the tested approaches over the period 1994 - 2016 (initial balance sheet F).

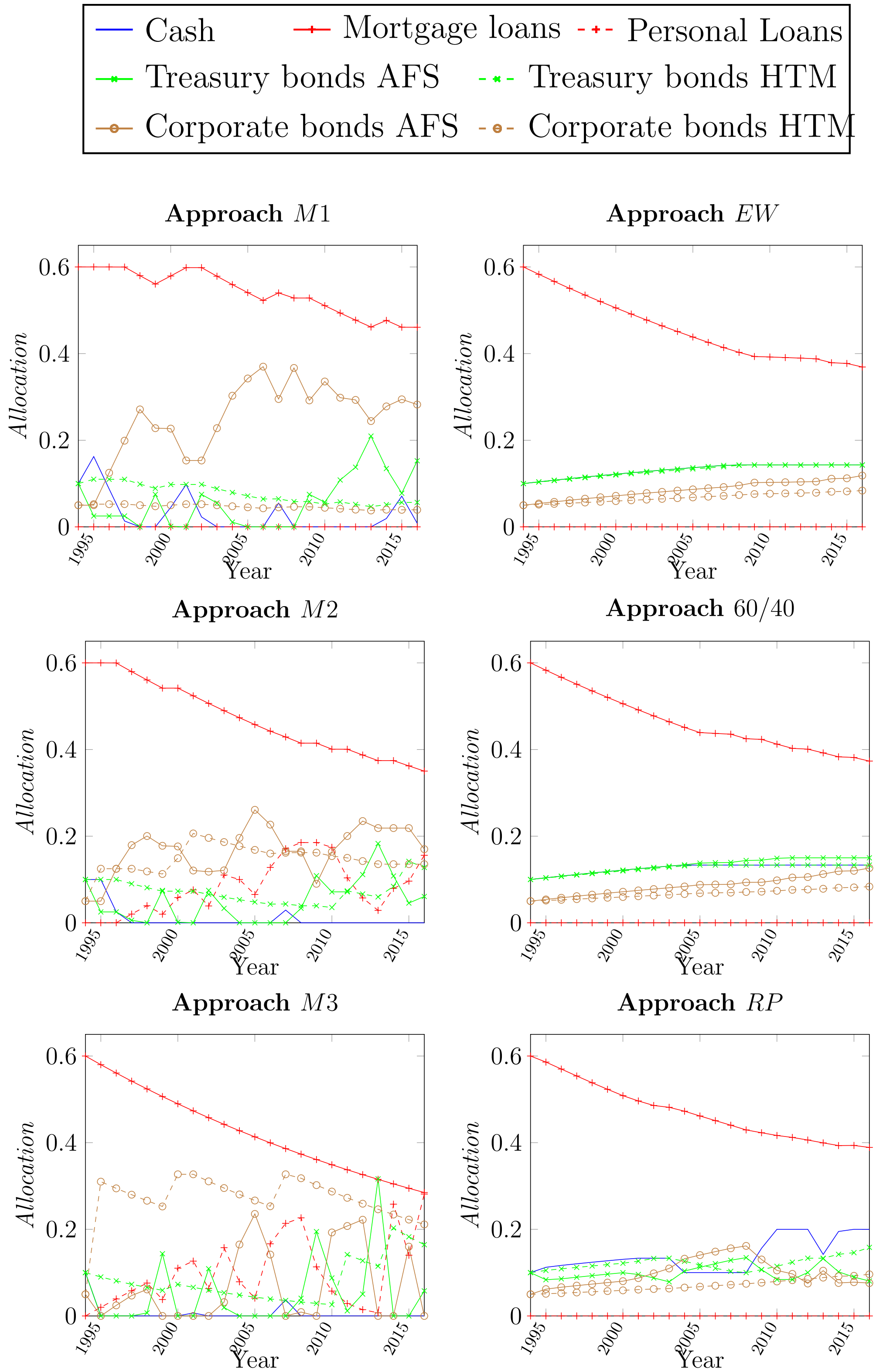

FIGURE 8. Evolution of the balance sheet during the simulation of the tested approaches over the period 1994 - 2016 (initial balance sheet G).