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SECONDARY MATHEMATICS FOR THE SOCIAL SCIENCES

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Abstract: The origins, rationale and development of the course *Mathematics Applied to the Social Sciences* (MACS), created in 2001, is described and some ideas around the teaching of mathematics in secondary schools for students other than the future scientists and engineers are discussed.

Keywords: Mathematics Education, Secondary Education, Social Sciences.
Math. Subject Classification (2000): 97B70 (97A80 97D30).

1. Introduction

There are two recurring debates about the mathematics curriculum in secondary schools, especially in countries like Portugal where compulsory education goes till the 12th grade. First, should all students study mathematics (not necessarily the same) or should the curriculum leave a part of the students *happy* without the mathematics *torture*? Second, should all students study the same *classic* mathematics, around ideas from differential and integral calculus with some Geometry and some Statistics?

When the 2001 revision (in great part in application today) of the Portuguese Secondary School curriculum was made (involving the 10th, 11th and 12th grades) different kinds of courses were introduced for the different tracks (but not for all of them) that traditionally existed. **Mathematics A** is for the Science and Technology track and for the Economics track and is a compulsory course. **Mathematics B** is for the Arts track and is an optional course. **Mathematics Applied to the Social Sciences** (MACS) is for the Social Sciences track and is an optional course. The Languages track was left without mathematics or science. Later the last two tracks were merged and

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the MACS course remained optional for the new merged track. The technological or professional tracks could have **Mathematics B**, **Mathematics for the Arts** or **Modules of Mathematics** (3 to 10 to be chosen from 16 different modules, depending on the professions).

2. THE ORIGINS OF THE MACS COURSE

The 1990 revision of the Portuguese Secondary School curriculum included for the first time a division of the Mathematics course into two different ones. **Mathematics** for some students and **Quantitative Methods** for others, including the students of the Arts track. This created quite a controversy at the time [30] because the syllabus of **Quantitative Methods** included Logic, Real Numbers, Functions and no Geometry or Applications (the course was offered only for the 10th grade students, so it was only one year long). Students that had this course normally did not like mathematics or were weak at it (but they had to *suffer* only 1 more year). This situation motivated a number of projects that tried to present alternatives, namely for Arts students, that included some Geometry [23] and proposals that included more modern topics like Graph Theory and Dynamical Systems [5]. João Pedro Ponte, a leading researcher in Mathematics Education in Portugal and a former member of the Board of ERME (European Society for Research in Mathematics Education), questioned whether a single Mathematics course for all students, centered on Pure Mathematics, that was necessary to enter Higher Education was or not one of the responsible for the very high retention rate. But there were no changes to the official curriculum till 2001 [24].

In 2001 there was a revision of the structure of secondary education, that defined seven different tracks with specific clear goals, but with the same structure: in each track there are 3 courses that are considered as foundational for that track and so are compulsory, there are several optional courses and a number of courses with an interdisciplinary flavor like *Project Area* (to develop project work) and ICT (Information and Communication Technology). This revision was implemented only in 2004, with some changes, but the general structure remained the same.

That's when it was proposed to have an optional mathematics course offered for the *Social Sciences* track. There were discussions about possibilities of having more courses (like *Mathematics in History and Philosophy* for the Languages track) for other tracks but finally MACS was chosen, only for the *Social Sciences* track, but not for other tracks (so, some tracks ended their

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part). Areas like Election Theory and Graph Models sparked a lot of interest and we now have quite a few publications. As stated by UNESCO *there cannot be any quality mathematics education for all unless quality resources are produced for pupils and for teachers* [29].

Several universities, namely the University of Coimbra, included in their Master degree topics to prepare future secondary school teachers like election theory, apportionment and graph theory (like the works supervised by Maria Celeste Gouveia [18]).

The Teacher Association APM, in its 2007 report said that *APM participated actively with proposals, teacher preparation, discussions, preparation of materials, etc.* and *the process (...) has been exemplary.* The authors of the MACS course had a permanent *contact with teachers in the field, asked for contributions from all the teachers, mathematicians and other specialists, integrated in a very satisfactory manner the several suggestions sent to them, and the authors also organized meetings to discuss the work being done in a very open way* [2].

With all this national movement and the positive reaction of students, we can say that there were conditions for the course to contribute positively to the success of Social Sciences students in Mathematics.

8. CONCLUSIONS

After 15 years there is no thorough evaluation of how the course is run in practice in the schools, or which is the real impact on the further education or activities of the students that studied **Mathematics Applied to the Social Sciences**. In Portugal there is no institution in charge of this type of work and evaluations are done on a case by case basis. All Secondary Schools need to do self-evaluations but normally just compare internal statistics to national ones to see where they are in the national scene. In the reports consulted there was no special mention to the MACS course and so we have the impression that the MACS course entered the normal Portuguese routine in Secondary School.

Now in Portugal compulsory education goes till the end of Secondary School, the 12th year. I hope we will evolve to some significant mathematics studies being offered to all kinds students at the secondary level, and not only to some students on a partially optional level, in order to guarantee a quality mathematics education for all, following some ideas expressed in the UNESCO document [29].

mathematical studies at the 9th grade level).

3. THE MACS COURSE

When, in 2001, there was a possibility to introduce a new Mathematics course for the *Social Sciences* track, for the 10th and 11th grade students, there were some discussions of what could be offered. The model chosen was inspired in the course *For All Practical Purposes* [10] developed by COMAP because it *uses both contemporary and classic examples to help students appreciate the use of math in their everyday lives.* As a consequence, a set of independent chapters, each one with some specific purpose, was chosen for this syllabus, that included 2 years of study, with 4.5 hours of classes per week (normally 3 classes of 90 minutes each). The topics chosen were:

10th grade
Decision Methods: Election Methods, Apportionment, Fair Division
Mathematical Models: Financial models, Population models
Statistics (regression)
11th grade
Graph models
Probability models
Statistics (inference)

The stated goal of this course is for the students to have *significant mathematical experiences that allow them to appreciate adequately the importance of the mathematical approaches in their future activities* [6], [7]. This means that the main goal is not to master specific mathematical concepts, but it is really to give students a new perspective on the real world with mathematics, and to change the students view of the importance that mathematical tools will have in their future life. In this course it is expected that the students study simple real situations in a form as complete as possible, and *develop the skills to formulate and solve mathematically problems and develop the skill to communicate mathematical ideas (students should be able to write and read texts with mathematical content describing concrete situations).*

This was a huge challenge for the Portuguese educational system because most of these topics had never been covered before, and most teachers did not

even study Graph Theory at University. Election Methods, Apportionment and Fair Division were of course completely new to everybody. The reception was good from the part of the Portuguese Math Teacher Association APM, as it considered that *the methodologies and activities suggested in the MACS program promote the development of the skills of social intervention, of citizenship and others* [2]. The reception from the scientific society SPM was rather negative because they considered the syllabus did not have enough mathematical content.

These new topics had in part been proposed previously. Back in 1942 the mathematician and educator Bento de Jesus Caraça complained with the topics of the secondary school syllabus that had nothing to do with *contemporaneous life* and where practical applications were absent [9]. Also, in 1994, the mathematics educator Paulo Abrantes wondered when topics like Graphs and Matrices would be introduced in our syllabuses, because they represent very different forms of mathematics reasoning [1].

4. THE CHOICE OF TOPICS

Being an optional course for secondary school students, the choice of topics is not constrained by further studies in Higher Education. The topics were chosen so that they could be used with secondary school students that normally are not a priority for mathematics studies, in order for them to encounter *significant mathematical experiences*. It is hoped that, although teachers may find difficulties implementing this program, they will achieve some satisfaction when they see that *students become aware how Mathematics is an important tool for their life* [6], [7].

Decision Methods were chosen because we live in a society where everybody is called to make decisions (for example in elections) and all need to be aware that mathematics gives some tools to choose an adequate method to arrive at a final decision.

The mathematical models are always incomplete but they can be useful to explain growth in a biological or economics situation, giving some information about when a population may become extinct. Graph models are useful to study in a more complete way systems of distribution or collection.

Probability and Statistics are so important in our times that they deserve to be discussed with some detail, and so these areas play an important part in this syllabus, including a new topic in any Portuguese Secondary School

There are some research studies that test different parts of the **MACS** syllabus. We will mention three of them. The first, done with the students of the 11th grade [20], concluded that students managed to study graphs in a problem solving environment and evolved in the knowledge of this area using *real world* problems. The second, done with students of the 10th grade in the topic of Population Models [17] concludes that students can do small investigations, work in groups with technology, communicate with others and understand how mathematics is used in the real world. The third was done again with students of the 11th grade but in the topic of probability models [26]; it uses technology to help students overcome difficulties with conditional probability. In all studies it is clear the role of *real world*problems.

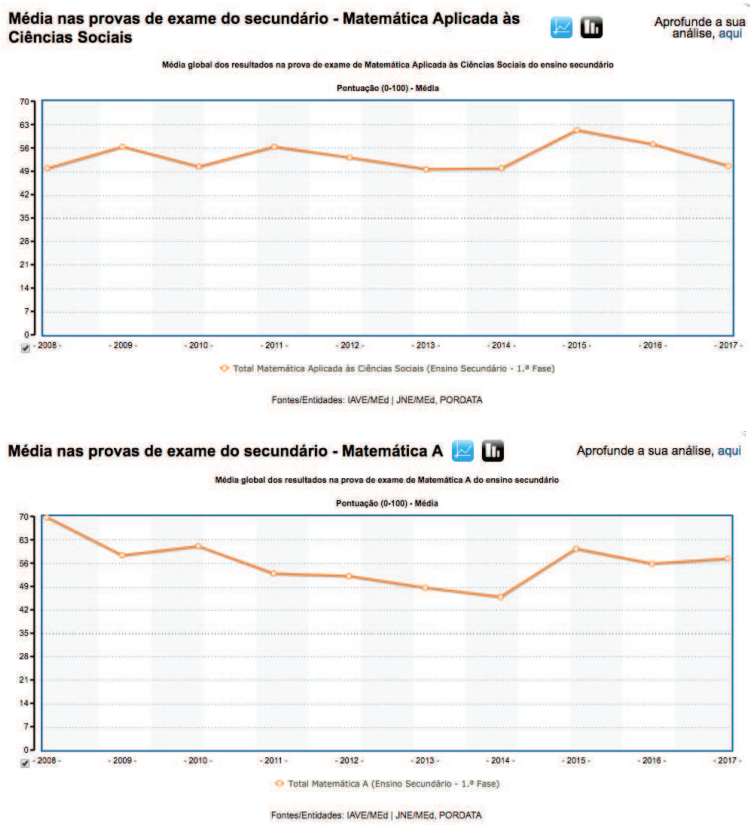
7. WHY THE MACS COURSE SURVIVES TILL TODAY

As **Quantitative Methods** failed as a discipline, there was skepticism that an *alternative* to mainstream mathematics could be feasible. No tradition in Portugal, no teachers prepared, no publications, no textbooks. But the fact is that a carefully designed plan allowed today's situation where thousands of students opt for this course and do a no high-stakes national examination on it. There are no calls to end it, and we can think that this course might be offered to other kinds of students with the same success ([3], [8], [21], [27], [28], [22], [12], [13], [14], [22]).

In 2001 there was a Consulting Committee at the General Directorate for Secondary Education that advised the Ministry on measures to be taken to improve the teaching of Mathematics at the Secondary School level. Under the guidance of this Committee, a set of Secondary School teachers was specially prepared in some working weeks, so that they would be prepared to make preparation of other teachers. Most of these teacher specialists in MACS produced professional development sessions for other teachers and so a way of preparing teachers to teach MACS was in fact functioning.

Written teaching materials were produced by the 3 authors of the MACS program. Several publications were made by several teachers, authorized translations of COMAP publications were edited by the Ministry of Education (including Election Theory, Apportionment and Graph Models), and new textbooks were produced (not without some trouble, one of the textbooks being taken out of the market due to serious errors in the statistics

The score obtained in the national **MACS** examination is normally higher than 50%, a little above the mean results of **Mathematics A** (source: *PORDATA*).



There was some controversy on the existence of this examination, as it apparently contradicted the stated goals of the course, but the results show that it did not seem to be a deterrent for a big number of students.

One clear topic winner of **MACS** among students was election theory. This topic clearly resonated in the student's experiences and there are numerous small projects connected to its use in very concrete situations, be it the parliamentary elections or the school's student union elections.

syllabus, the Statistical Inference, to show how scientific conclusions can be inferred from sets of data.

Other topics were proposed like game theory and cryptography, but no more topics were chosen so that teachers and students would have time to explore the syllabus, namely exploring concrete situations, look for data, develop some projects, explore the History of Mathematics (like the Konigsberg Bridges problem) and use relevant technology (graphing calculators being compulsory).

5. THE NATIONAL EXAMINATION

A very controversial matter about the MACS course is the existence of a national examination that counts 30% for the final grade of the student in the course. The present regulations state that students need to do a total of 4 national examinations in order to be granted the Secondary School diploma.

APM points out that the existence of the national examination is not compatible with the assessment suggested in the official syllabus [2]. The association complains that teachers lose their freedom and try to *prepare* students for the examination and this somehow does not allow the innovation aspects of this program to pass fully into practice. In fact, the official syllabus of MACS gives a great freedom to the teacher to use a number of assessment instruments and recommends to not give priority to timed tests. Group work and individual work is recommended, assuming different forms: essays, personal notes, reports, presentations, debates. With a national examination, teachers complain they would need a more detailed specification of what is covered by the examination, but if the questions remain very open it is not feasible to give these kinds of very precise details. The first year the examination was administered the authors of the program prepared 3 model examinations to guide teachers (it was chosen not to produce only 1 in order to give a more open view of possible questions and not introduce unintended limitations on the format of questions).

Today the national exam of MACS consists of several rather mostly open but simple questions, where some careful interpretation or model construction/analysis is required. We give two examples from the 2017 examination. The first involves an exponential model that needs to be compared with a previously studied logarithmic model, using a graphing calculator.

5.2. A empresa *ComPromo* disponibilizou uma bilheteira *online*, na qual também é possível comprar bilhetes para o parque de diversões. As duas bilheteiras entraram em funcionamento no mesmo instante.

Admita que o número total de bilhetes vendidos pela bilheteira disponibilizada pela *ComPromo*, ao fim de t dias após a sua abertura, é bem aproximado pelo modelo seguinte, com arredondamento às unidades.

$$c(t) = 35e^{0,14t}, \text{ com } 0 < t < 30$$

Ao fim de quantos dias, após a abertura das duas bilheteiras, o número total de bilhetes vendidos na bilheteira *online* do parque foi, pela primeira vez, inferior ao número total de bilhetes vendidos na bilheteira disponibilizada pela *ComPromo*?

Para responder a esta questão, recorra às capacidades gráficas da sua calculadora e apresente:

– o(s) gráfico(s) visualizado(s) que lhe permite(m) resolver o problema;

– as coordenadas do(s) ponto(s) relevante(s) arredondadas às décimas.

The second involves applying a given voting method to a concrete situation.

Tabela 2

N.º de votos / Preferências	602	309	727
1.ª	C	A	D
2.ª	B	B	B
3.ª	A	C	C
4.ª	D	D	A

Concluída a votação, o apuramento da ementa vencedora é feito através do método a seguir descrito.

- Seleciona-se um par de ementas e atribui-se o número de votos registados em cada coluna à ementa mais bem posicionada, de entre as duas selecionadas.
- Comparam-se os votos obtidos por essas duas ementas. A ementa com o maior número de votos é a vencedora do par escolhido.
- Repetem-se os procedimentos anteriores até uma das ementas ter vencido em todas as comparações com as restantes. Essa ementa é a vencedora.

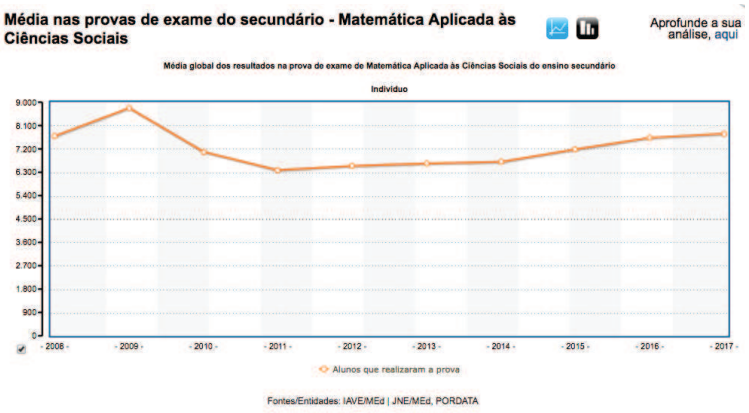
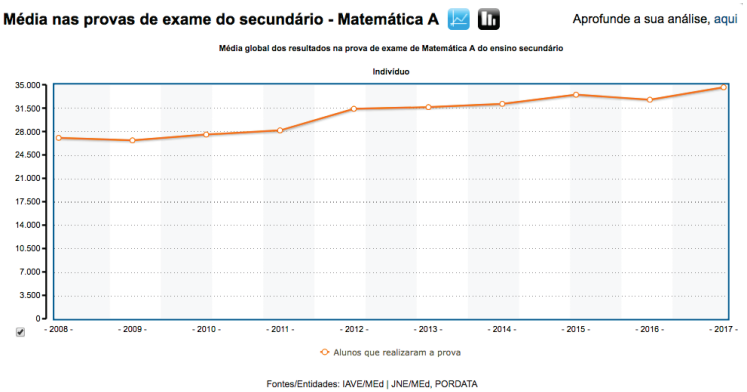
The marks students usually get on this national examination are similar to the results of other courses.

6. IS THE MACS COURSE A SUCCESS?

Being an optional course for secondary school students, and as mathematics is not very popular in Portugal, one would think that this course does not attract many students.

As this course is accepted by very few Higher Education degrees, students that take this course can easily opt not to take the National examination.

The number of students that take this examination is in fact very high. The total number of students taking exams is around 50 000, and some 30 000 take the main **Mathematics A** examination (source: *PORDATA*).



The number of students taking the **MACS** exam is around 8 000, which means around 25% of the students taking **Mathematics A**, higher than most people would expect (source: *PORDATA*).