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Abstracts

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9:00-9:15 Opening session

9:15-10:05 Rolf Sören Krauhssar, Katholieke Universiteit Leuven, Belgium

Applications of Clifford analysis methods to Maxwell and Helmholtz type equations on spheres, cylinders and on tori in higher dimensions

(partially joint work with I. Cacao (Universidade de Aveiro) and D. Constales (Ghent University))

In this talk we use recent methods from Clifford analysis to develop explicit representation formulas for the solutions to the time harmonic Maxwell equations, the homogeneous Helmholtz and Klein Gordon type equations on spheres, on conformally flat cylinder and on the tori associated to several different spinor bundles. Then we also consider inhomogeneous Helmholtz and Klein type Gordon equations with prescribed boundary data on these manifolds.

10:15-10:40 Hendrik De Bie, University of Ghent, Belgium

Generating monogenic functions in orthogonal Clifford analysis and extensions

An important problem in Clifford analysis is the construction of monogenic functions, i.e. nullsolutions of the Dirac operator. At present, several techniques are available, some of the most important ones being Cauchy-Kowaleskaia extensions, Fueter type theorems and (bi-)axial systems (giving rise to Vekua systems). In this talk we first give an overview of the (recent) literature on these techniques in orthogonal Clifford analysis. Next we will discuss the extension of these techniques to the case of the super Dirac operator, which gives rise to new sets of solutions and sometimes singular behaviour.

10:45-11:15 Coffee break

11:15-11:40 Dixan Peña- Peña, University of Aveiro, Portugal

Extensions theorems for holomorphic and biregular functions

(joint work with R. Abreu Blaya, J. Bory Reyes and F. Sommen)

In this communication, we discuss an integral representation formula for the so-called isotonic functions. We also define the isotonic Cauchy transform and present the Sokhotski-Plemelj formulae for this integral operator. Finally, as the holomorphic and the biregular functions satisfy the isotonic system, we use our results to obtain extension theorems for these types of functions.

11:45-12:10 Dimitris A. Pinotsis, University of Reading, U.K.

On the Physical Significance of the $\overline{D}$ Formalism and Quaternions

We first revisit Quaternionic Analysis in R^3 and discuss a representation of a quaternion-valued function in terms of two real harmonic functions. Then, we present certain integral representations of the solutions of elliptic Partial Differential Equations (PDEs) in three dimensions, such as the Poisson, inhomogeneous Biharmonic and Navier equations, and consider some relevant boundary value problems. Also, we present a physical application of an important problem in complex and quaternionic analysis called the $\overline{D}$ problem, thereby elucidating the physical significance of the associated formalism.

12:15-12:40 Nelson Faustino, University of Aveiro, Portugal

Rediscovering Clifford analysis: on the interplay between Umbral Calculus and Quantum Mechanics

Umbral calculus in its modern form (see [5]) has been revealed an alternative tool for calculations with polynomials. Applications of the umbral calculus cover several branches of mathematics like combinatorics, special function theory and approximation theory. In the present setting, the algebra of multivariate polynomials $\mathbb{R}[\underline{x}]$ can be realized as the free algebra generated by position and momentum operators, x'_j and O_{x_j}, respectively, satisfying the Heisenberg-Weyl commutation relations

$$[O_{x_j}, O_{x_k}] = 0, \quad [x'_j, x'_k] = 0, \quad [O_{x_j}, x'_k] = \delta_{jk}\mathbf{id}$$

makes fully rigorous the description of umbral calculus in terms of boson calculus associated to the second quantization approach [2]. In this talk we will establish the bridge between Clifford Analysis and the physical model of the harmonic oscillator encoded by the Hamiltonian operator

$$\mathcal{H}' = \sum_{j=1}^{n} \left(\frac{1}{2}(x'_j)^2 - \frac{1}{2}O^2_{x_j} \right)$$

consisting of a chain of n independent one-dimensional Hamiltonians, each of them having the unit mass and unit frequency. The approach that we will consider is motivated by the observation that classes of Wigner Quantum systems are canonically equivalent to the Lie superalgebra $osp(1|2n)$ (c.f. [4]) and from the fact that Clifford Analysis in its minimal form corresponds to a realization of the Lie superalgebra $osp(1|2)$ (c.f. [1]). As we will see along this talk, the former approach unifies continuous and discrete Clifford Analysis [3] as isomorphic quantal systems intertwined by a gauge transformation.

References

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Lunch

14:30-15:20 Kenneth McLaughlin, University of Arizona, U.S.A.

Some classes of random Hermitian matrices: $F(Tr(V(M))$ **instead of** $Tr(V(M))$

I will present two or three examples. We'll start with the usual random Hermitian matrices as an introduction. Then we'll consider two examples of probability measures on random Hermitian matrices which are invariant ensembles but which have peculiar, and different behavior. In all three cases, the primary goal will be to summarize explicit formulae for eigenvalue statistics, and with remaining time, discuss the subsequent asymptotic analysis.

15:30-15:55 Ulises Fidalgo Prieto, University Carlos III of Madrid, Spain

Nikishin systems are perfect

Perfectness in Nikishin systems is proved.

16:00-16:25 Maria das Neves Rebocho, University of Beira Interior, Portugal

On second order differential equations for orthogonal polynomials on the unit circle

(joint work with A. Branquinho (University of Coimbra, Portugal))

We present a study on sequences of orthogonal polynomials on the unit circle whose corresponding Carathéodory function satisfies a Riccati-type differential equation with polynomial coefficients. It is given a characterization of such polynomials in terms of matrix second order differential equations. In the semi-classical case, a characterization in terms of second order linear differential equations with polynomial coefficients is deduced.

16:30-16:55 Márcio Nascimento , Escola Superior de Tecnologia de Viseu, Portugal

On linearly related sequences of derivatives of orthogonal polynomials

(joint work with José Carlos Petronilho (University of Coimbra, Portugal))

We discuss an inverse problem in the theory of (standard) orthogonal polynomials involving two orthogonal polynomial families $(P_n)_n$ and $(Q_n)_n$ whose derivatives of higher orders m and k (resp.) are connected by a linear algebraic structure relation such as $\sum_{i=0}^{N} r_{i,n}\, P^{(m)}_{n-i+m}(x) = \sum_{i=0}^{M} s_{i,n} Q^{(k)}_{n-i+k}(x)$ for all $n = 0, 1, 2, \cdots$, where M and N are fixed non-negative integer numbers, and $r_{i,n}$ and $s_{i,n}$ are given complex parameters satisfying some natural conditions. Let $\mathbf{u}$ and $\mathbf{v}$ be the moment regular functionals associated with $(P_n)_n$ and $(Q_n)_n$ (resp.). Assuming $0 \leq m \leq k$, we prove the existence of four polynomials Φ_{M+k+i} and Ψ_{N+k+i}, of degrees $M+k+i$ and $N+k+i$ (resp.), such that $D^{k-m}\left(\Phi_{M+k+i}\mathbf{u}\right) = \Psi_{N+k+i}\mathbf{v} \quad (i = 0, 1)$, the $(k-m)$th$-$derivative, as well as the left-product of a functional by a polynomial, being defined in the usual sense of the theory of distributions. If $k = m$, then $\mathbf{u}$ and $\mathbf{v}$ are connected by a rational modification. If $k = m + 1$, then both $\mathbf{u}$ and $\mathbf{v}$ are semiclassical linear functionals, which are also connected by a rational modification. When $k > m$, the Stieltjes transform associated with $\mathbf{u}$ satisfies a non-homogeneous linear ordinary differential equation of degree $k - m$ with polynomial coefficients.

17:00-17:30 Coffee break

17:30-17:55 Svend Ebert, University of Freiberg, Germany

Wavelets on S^n

In the talk I would like to discuss the construction of Wavelets on the n-dimensional sphere S^n. The task to find a square integrable, irreducible representation of $SO(n+1,1)$ in $L^2(S^n)$ we split into the the regular representation of rotations on the one hand and call in the convolution with an approximate identity as dilation one the other hand. Since for the wavelet transformation of nonzonal wavelets we need to work with an orthogonal system, we introduce the system of orthogonal polynomials in $L^2(SO(n+1))$ similar to Wigner polynomials on $L^2(SO(3))$.

18:00-18:25 Ming-Guang Fei, University of Aveiro, Portugal

Direct Sum Decomposition of $L^2(\mathbf{R}_1^n)$ into Subspaces Invariant Under Fourier Transformation

Denote by $\mathbf{R}_1^n$ the real-linear span of $\mathbf{e}_0, \mathbf{e}_1, ..., \mathbf{e}_n$, where $\mathbf{e}_0 = 1, \mathbf{e}_i\mathbf{e}_j + \mathbf{e}_j\mathbf{e}_i = -2\delta_{ij}$. Under the concept of left-monogeneity defined through the generalized Cauchy-Riemann operator we obtain the direct sum decomposition of $L^2(\mathbf{R}_1^n)$, $n \geq 1$,

$$L^2(\mathbf{R}_1^n) = \sum_{k=-\infty}^{\infty} \bigoplus \Omega^k,$$

where Ω^k is the right-Clifford module of finite linear combinations of functions of the form $R(x)h(|x|)$, where, for $d = n+1$, the function R is a k- or $-(d+|k|-2)$-homogeneous left-monogenic function, for $k > 0$ or $k < 0$, respectively, and h is a function defined on $[0, \infty)$ satisfying a certain integrability condition in relation to k. The spaces Ω^k are invariant under Fourier transform. This extends the classical result for $n = 1$. We also deduce explicit Fourier transform formulas for functions of the form $R(x)h(r)$ refining Bochner's result for spherical k-harmonics.

Saturday, 18

9:00-9:50 Arno Kuijlaars, Katholieke Universiteit Leuven, Belgium

Random matrices in an external source

In this joint work with Pavel Bleher we study the Hermitian nxn random matrix model with potential V and external source with two eigenvalues -a and +a of equal multiplicity. The eigenvalue correlations are described by multiple orthogonal polynomials for two exponential weights $exp(-n(V(x)-ax))$ and $exp(-n(V(x)+ax))$. For a quadratic potential the model is equivalent to a model of non-intersecting Brownian paths. In the large n limit there is a phase transition described by Pearcey integrals. We show that this is a non-generic phenomenon. For a non-quadratic potential V the Pearcey phase transition does not occur, but instead there are two phase transitions of Painlevé II type.

10:00-10:25 Steven Delvaux, Katholieke Universiteit Leuven, Belgium

A graph-based equilibrium problem for the limiting distribution of non-intersecting Brownian motions

Consider n non-intersecting Brownian motions with fixed starting positions at time $t = 0$ and fixed ending positions at time $t = 1$. The numbers of distinct starting and ending positions may be arbitrary. We show that if the temperature T is sufficiently small, then the limiting distribution of the Brownian particles in the large n limit can be characterized in terms of a vector equilibrium problem. The form of the equilibrium problem depends on a graph in a similar way as the equilibrium problems studied by A.A. Gonchar, E.A. Rakhmanov, and V.N. Sorokin. Our proof is based on a Riemann-Hilbert steepest descent analysis.

10:30-10:55 Joris Verbaenen, Katholieke Universiteit Leuven, Belgium

Theorems of Rakhmanov and Denisov about Recursion Coefficients of Orthogonal Polynomials

A set of orthonormal polynomials on the real line satisfy a three term recurrence relation. From Weyl's theorem we already know that if the recurrence coefficients converge to 1/2 and 0, then the absolute continuous part of μ has support on $[-1, 1]$.

For my master thesis I study the converse of Weyl's theorem. Unfortunately, no new results have been found by me. A partial convers was given by Rakhmanov (1982) in the case μ has support on $[-1, 1]$. Later, in 2002, Denisov proved the converse for Weyl's theorem under the more general condition that the support of the measure μ could have a set of singular points E outside of $[-1, 1]$.

11:00-11:20 Coffee break

11:20-11:45 Klaas Deschout, Katholieke Universiteit Leuven, Belgium

A phase transition for Jacobi-Angelesco polynomials

(joint work with Arno B. J. Kuijlaars)

We consider Jacobi-Angelesco polynomials. These are multiple orthogonal polynomials associated to two Jacobi weights on touching intervals $[a, 0]$ and $[0, 1]$. We show that for $a = -1$, there is a phase transition. The behavior of the Jacobi-Angelesco polynomials in this phase transition can be studied using a Riemann-Hilbert problem (RHP). The idea is to transform the original RHP into a RHP for which one has good estimates by means of the well known Deift-Zhou steepest descent method. A crucial step in this method is the construction of a local parametrix P around 0. The function P can be constructed using solutions of a rescaled version of the differential equation for Jacobi-Angelesco polynomials. However, one may also exploit a connection with a distinct problem: the behavior of non-intersecting squared Bessel paths around the critical time.

11:50-12:15 Dries Geudens, Katholieke Universiteit Leuven, Belgium

An equilibrium problem for the two matrix model in the quartic/quadratic cases

Associated with the two matrix model with potentials V and W are two sequences of biorthogonal polynomials. In a recent article, Duits and Kuijlaars gave a steepest descent analysis of the 4 by 4 matrix-valued Riemann-Hilbert problem that characterizes one of the biorthogonal polynomials for the special case $W(y) = y^4/4$. An important ingredient in their analysis is an equilibium problem for a vector of three measures.

Here we will show how the vector equilibrium problem arises for the specific choice of a quadratic potential $V(x) = x^2/2$ but for a more general quartic potential $W(y) = y^4/4 + ty^2/2$.

Our approach is based on the limiting behavior of the recurrence coefficients of the biorthogonal polynomials. Our analysis reveals that a new criticial behavior occurs for certain negative values of t.

12:20-12:45 Pablo Roman, Katholieke Universiteit Leuven, Belgium

Asymptotic analysis of recurrence coefficients coming from non-intersecting squared Bessel paths

(Joint work with Arno Kuijlaars)

In this work we consider a model of n non-intersecting squared Bessel paths conditioned so that all paths start at time $t = 0$ at the same positive value $a = 0$, remain positive and end at time $t = T$ at $x = 0$. The positions of the paths at any time form a multiple orthogonal polynomial ensemble corresponding to a system of two modified Bessel-type weights. The type II multiple orthogonal polynomials associated with these weights satisfy a four term recurrence relation. We associate to it a family of $n \times n$ banded Toeplitz matrices $\{T_n^s\}_{s \geq 0}$. The limiting normalized eigenvalue distribution of T_n^s, μ_0^s, is the first component of (μ_0^s, μ_1^s) the unique vector of measures that minimizes an energy functional. By integrating the Euler-Lagrange equations for the equilibrium problem for (μ_0, μ_1), we obtain an equilibrium problem for (ν_0, ν_1),

$$\nu_0 = \int_0^1 \mu_0^s ds, \quad \nu_1 = \int_0^1 \mu_1^s ds.$$

This is an equilibrium problem with an external field, and an upper constraint acting on ν_1.

Lunch

14:30-14:55 Franciscus Sommen, University of Ghent, Belgium

Clifford Calculus in Quantum Variables

Starting from the axiomas of the radial algebra together with the basic q-commutation relations for the coordinates: $x^i x^j = q_{i_j} x^j x^i$ we arrive at the defining relations for the q-Clifford algebra: $e_i e_j + q_{j_i} e_j e_i = -2g_{i_j}$, whereby g_{i_j} is the q-metric which also consists of non-commuting parameters. The partial derivatives ∂_{x_j} satisfy the same q-relations $\partial_{x_i}\partial_{x_j} = q_{i_j}\partial_{x_j}\partial_{x_i}$ together with the q-Weyl relations $\partial_{x_i} x^j = q_{j_i} x^j \partial_{x_i} + \delta_{i_j}$. This leads to the introduction of the reciprocal Clifford basis e^j satisfying $e_j e^i + q_{j_i} e^i e_j = -2\delta_{i_j}$ which is linked to the original Clifford basis by relations of the form $e_j = g_{j_k} e^k$ (assuming the Einstein-summation convention). The vector derivative (Dirac operator) is then given by $\partial_x = \partial_{x_j} e^j$ and the basic rules of Clifford calculus can be derived. In the level of radial algebra thes rules are the same as for standard Clifford analysis, which indicates that the q-deformation aspect is only visible when calculations are expressed in coordinates.

15:00-15:25 Milton Ferreira, Polytechnical Institute of Leiria, Portugal

Gabor analysis over the rotation group

This talk is concerned with the problem of the inversion of the one-dimensional Radon transform on the rotation group $SO(3)$ and its application to X-Ray tomography with polycrystalline materials. The proposed approach is composed by Gabor frames constructed through the coorbit theory on homogeneous spaces, and variational principles for sparse reconstructions that yield iterative approximation of the solution of the inverse problem. linear independent solutions of this problem is obtained.

15:30-15:55 Rui Marreiros, University of Algarve, Portugal

On an estimate for the number of solutions of a generalized Riemann boundary value problem with shift.

(joint work with Viktor G. Kravchenko and Juan C. Rodriguez)

In the real space of all Lebesgue measurable complex valued functions on $\mathbb{R}$, which are absolutely integrable in the second power, $\widetilde{L}_2(\mathbb{R})$, we consider the generalized Riemann boundary value problem: find the functions $\varphi_+(z)$ and $\varphi_-(z)$ analytic in the upper halfplane and in the lower halfplane, respectively, satisfying the condition $\varphi_+ = a\varphi_- + b\varphi_-(\alpha) + c\overline{\varphi_-} + d\overline{\varphi_-(\alpha)}$, $\varphi_-(\infty) = 0$, imposed on their boundary values, i. e., on $\mathbb{R}$, where $\alpha(t) = t + h$, $h \in \mathbb{R}$, is the shift on the real line, and a, b, c, d are continuous functions on $\overset{\circ}{\mathbb{R}} = \mathbb{R} \cup \{\infty\}$, the one point compactification of $\mathbb{R}$. Under certain conditions on the coefficients, an estimate for the number of linear independent solutions of this problem is obtained.

16:00-16:25 Raúl Castillo, University of Aveiro, Portugal

An implementation of pseudoanalytic functions for the design of optical filters

An implementation of pseudoanalytic functions for the design of optical filters The accuracy of a recently developed numerical method [1] which allows to obtain the reflectance and transmittance coefficients for an inhomogeneous layer is used to characterize optical filters in terms of some of their most important parameters (e. g. center wavelength, bandwidth, Mode Suppression Ratio (MSR) and Full Width at Half Maximum (FWHM)). The method is based on new representations for solutions of the Sturm-Liouville equations obtained in [2] and developed at [3]. The solution there is represented in the form of a functional series. Tailoring the index profile makes it possible to modify some of the characteristics of the filters. Different profiles are evaluated with the aid of Matlab (like linear, exponential sinusoidal and chirped).

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16:30-17:00 Coffee break

17:00-17:25 Andrei Martinez Finkelstein University of Almeria, Spain

Critical measures and some applications

The concept of critical measure generalizes the idea of an equilibrium measure with the symmetry property, which finds well known applications in the asymptotic analysis of polynomials of complex orthogonality and Padé approximants to algebraic functions. Here we discuss the definition of a critical measure in an external field, it characterization, and an application to the description of the asymptotic behavior of the so-called Heine-Stieltjes polynomials (polynomial solutions of certain second order ODEs).

17:30-17:55 Vitor Sousa, University of Aveiro, Portugal

Asymptotics of orthogonal polynomials for a weight with a jump

(joint work with: A. Foulquié Moreno and A. Martínez-Finkelshtein)

We consider the orthogonal polynomials on $[-1, 1]$ with respect to the weight

$$w_c(x) = h(x)(1-x)^{\alpha}(1+x)^{\beta}\,\Xi_c(x), \quad \alpha, \beta > -1,$$

where h is real analytic and strictly positive on $[-1, 1]$, and Ξ_c is a step-like function: $\Xi_c(x) = 1$ for $x \in [-1, 0)$ and $\Xi_c(x) = c^2$, $c > 0$, for $x \in [0, 1]$. We study the asymptotics of the Christoffel-Darboux kernel in a neighborhood of the jump and show that the zeros of the orthogonal polynomials no longer exhibit the clock behavior.

For the asymptotic analysis we use the steepest descendent method of Deift and Zhou applied to the non-commutative Riemann-Hilbert problems characterizing the orthogonal polynomials. The local analysis at $x = 0$ is carried out in terms of the confluent hypergeometric functions, and, at this point leads to a different kernel constructed in terms of this functions.

18:00-18:25 Cristina Diogo, University of Minho, Portugal

Corona conditions and symbols with a gap around zero

It is known that if a bounded analytic solution to the Riemann-Hilbert problem $Gh_+ = h_-$ is known and satisfies the corona conditions, then G admits a canonical generalized factorization, i.e., the Toeplitz operator with symbol G is invertible. But what if the solution does not satisfy the corona conditions? And, to begin at the beginning, how to get a particular bounded analytic solution to the Riemann-Hilbert problem? These questions are adressed by studying a class of triangular matrix symbols which illustrates the problems involved and for which we can find answers.