1. Cálculo Diferencial

Limite, continuidade e diferenciabilidade de funções vectoriais de várias variáveis.

1. (a) $F(x,y)=(F_1(x,y),F_2(x,y))$, com $F_1(x,y)=-y/(x^2+y^2)$, $F_2(x,y)=x/(x^2+y^2)$;
(b) $D_F=D_{F_1}\cap D_{F_2}=\mathbb{R}^2\setminus\{(0,0)\}$; (c) O domínio de continuidade de F é $\mathbb{R}^2\setminus\{(0,0)\}$;
(d) Matriz jacobiana de F, $J_F(x,y)=\begin{bmatrix}2xy & y^2-x^2\\ y^2-x^2 & -2xy\end{bmatrix}/(x^2+y^2)^2$, $(x,y)\neq(0,0)$.

2. (a) $J_f(x,y)=\begin{bmatrix}\sin y & x\cos y\\ y^2e^x & 2ye^x\end{bmatrix}$; (b) $J_g(x,y,z)=\begin{bmatrix}2x & 2y & 2z\\ z\cos(xz) & 0 & x\cos(xz)\end{bmatrix}$;
(c) $J_h(t)=\begin{bmatrix}-\sin t\\ \cos t\end{bmatrix}$; (d) $J_\varphi(u,v,w)=\begin{bmatrix}e^u\cos v\sin w & -e^u\sin v\sin w & e^u\cos v\cos w\\ e^u\sin v\sin w & e^u\cos v\sin w & e^u\sin v\cos w\\ e^u\cos w & 0 & -e^u\sin w\end{bmatrix}$.

3. (a) $J_F(r,\theta)=\begin{bmatrix}\cos\theta & -r\sin\theta\\ \sin\theta & r\cos\theta\end{bmatrix}$; (b) $J_G(\rho,\theta,\phi)=\begin{bmatrix}\cos\theta\sin\phi & -\rho\sin\theta\sin\phi & \rho\cos\theta\cos\phi\\ \sin\theta\sin\phi & \rho\cos\theta\sin\phi & \rho\sin\theta\cos\phi\\ \cos\phi & 0 & -\rho\sin\phi\end{bmatrix}$;
$\det(J_F(r,\theta))=r$; $\det(J_G(\rho,\theta,\phi))=-\rho^2\sin\phi$.

4. $D_ug(\pi,-2,1)=J_g(\pi,-2,1)\cdot u=[6\,\pi+20\ \ -3]^t$; $D_u\varphi(0,\pi/4,\pi/4)=J_\varphi(0,\pi/4,\pi/4)\cdot u$
$=[4\ \ -1\ 3/\sqrt{2}\,]^t$. 5. $J_f(P_0)=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 1\end{bmatrix}^t$; $D_{\hat u}f(P_0)=J_f(P_0)\cdot\hat u=[1/\sqrt{2}\ \ -1/\sqrt{2}\ \ -1/\sqrt{2}]^t$.

Derivada de funções compostas.

6. $u_x(x,y,z)=3x^2\,F(y/x,z/x)-x\,y\,F_a(y/x,z/x)-x\,z\,F_b(y/x,z/x)$; $u_y(x,y,z)=x^3\,F_a(y/x,z/x)$;
$u_z(x,y,z)=x^3\,F_b(y/x,z/x)$. 7. $z_x(x,y)=-1/x^2\,\phi'(1/x+\ln y)$; $z_y(x,y)=y+1/y\,\phi'(1/x+\ln y)$.
8. (a) $g_v(u,v)=\dfrac{\partial}{\partial v}[f(1/u^2)]=0$; (b) $g_u(1,0)=-4$. 9. –

10. (a) $w_x(x,y,z)=-f_b(y-z,z-x,x-y)+f_c(y-z,z-x,x-y)$; $w_y(x,y,z)$
$=f_a(y-z,z-x,x-y)-f_c(y-z,z-x,x-y)$; $w_z(x,y,z)=-f_a(y-z,z-x,x-y)$
$+f_b(y-z,z-x,x-y)$.
(b) $\dfrac{\partial^2 w}{\partial y\,\partial x}(x,y,z)=-f_{ba}(y-z,z-x,x-y)+f_{bc}(y-z,z-x,x-y)+f_{ca}(y-z,z-x,x-y)$
$-f_{c^2}(y-z,z-x,x-y)$; $\dfrac{\partial^2 w}{\partial x\,\partial z}(x,y,z)=f_{ab}(y-z,z-x,x-y)-f_{ac}(y-z,z-x,x-y)$
$-f_{b^2}(y-z,z-x,x-y)+f_{bc}(y-z,z-x,x-y)$.

11. (a) $\dfrac{\partial F}{\partial x}(x,y)=2\,g_u(2x,y^2)/g(2x,y^2)$; $\dfrac{\partial F}{\partial y}(x,y)=2\,y\,g_v(2x,y^2)/g(2x,y^2)$. (b) –

12. (a) $\dfrac{\partial f}{\partial x}(x,y) = g(x+y)\,;\ \dfrac{\partial f}{\partial y}(x,y) = g(x+y)\,;$

(b) $\dfrac{\partial f}{\partial x}(x,y,z) = g(\sin(x\,\sin(y\,\sin z)))\,\sin(y\,\sin z)\,\cos(x\,\sin(y\,\sin z)) - 4\,x^3\,g(x^4)\,;$

$\dfrac{\partial f}{\partial y}(x,y,z) = g(\sin(x\,\sin(y\,\sin z)))\,\cos(x\,\sin(y\,\sin z))\,x\,\sin z\,\cos(y\,\sin z)\,;$

$\dfrac{\partial f}{\partial z}(x,y,z) = g(\sin(x\,\sin(y\,\sin z)))\,\cos(x\,\sin(y\,\sin z))\,x\,y\,\cos z\,\cos(y\,\sin z)\,.$

13. $\Delta f(r\cos\theta, r\sin\theta) = \dfrac{\partial^2 g}{\partial r^2}(r,\theta) + \dfrac{1}{r^2}\dfrac{\partial^2 g}{\partial \theta^2}(r,\theta) + \dfrac{1}{r}\,.$

14. $\Delta f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)$

$= \dfrac{\partial^2 g}{\partial \rho^2}(\rho,\theta,\phi) + \dfrac{1}{\rho^2}\dfrac{\partial^2 g}{\partial \phi^2}(\rho,\theta,\phi) + \dfrac{1}{\rho^2\sin^2\phi}\dfrac{\partial^2 g}{\partial \theta^2}(\rho,\theta,\phi) + \dfrac{2}{\rho}\dfrac{\partial g}{\partial \rho}(\rho,\theta,\phi) + \dfrac{1}{\rho^2\tan\theta}\dfrac{\partial g}{\partial \phi}(\rho,\theta,\phi)\,.$

15. $J_F(0,0) = \nabla F(0,0) = \begin{bmatrix} 1 & 0 \end{bmatrix}.$ 16. –

17. f é de classe C^∞ em $\mathbb{R}^4$ e g é de classe C^∞ em $\mathbb{R}^2$; logo $g\circ f$ é de classe C^∞ em $\mathbb{R}^4$ portanto, diferenciável em $\mathbb{R}^4$ e $J_{g\circ f}(1,1,1,1) = J_g(2,2)\,J_f(1,1,1,1) = \begin{bmatrix} 4\,e^4 & 4\,e^4 & 4\,e^4 & 4\,e^4 \\ 16 & 8 & 16 & 8 \end{bmatrix}.$

18. $J_f(x,y) = \nabla f(x,y) = \begin{bmatrix} 2x\cos(x^2-y^2) & -2y\cos(x^2-y^2) \end{bmatrix};\ J_g(x,y) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix};$

$J_{f\circ g}(x,y) = \begin{bmatrix} 4y\cos(4xy) & 4x\cos(4xy) \end{bmatrix}.$

19. $J_{g\circ f}(x,y,z) = \begin{bmatrix} 2x+2y^2+y^2z & 4xy+4y^3+2xyz & xy^2 \\ 2xy^2z+y^4z & 4xy^3z+2x^2yz & x^2y^2+xy^4 \\ y^2ze^{xy^2z} & 2xyze^{xy^2z} & xy^2e^{xy^2z} \end{bmatrix}.$

20. $J_{h\circ g}(1,1,2) = \begin{bmatrix} -36 & 0 & 6 \end{bmatrix}.$ 21. $J_F(1,\pi/4,\pi/2) = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & -2 \\ \sqrt{2} & 0 & -1 \end{bmatrix}.$

22. (a) –; (b) Se $v = (v_1,v_2) \neq (0,0)$ então $\dfrac{\partial f}{\partial v}(0,0) = v_1\,v_2^2(v_1^2+v_2^2)\,;\ \dfrac{\partial f}{\partial x}(0,0) = \dfrac{\partial f}{\partial y}(0,0) = 0\,.$
(c) Não. (d) –

Teorema da função implícita. Teorema da função inversa. Dependência funcional.

23. (a) $(x_0, x_0+\sqrt{x_0^2+1})$ ou $(x_0, x_0-\sqrt{x_0^2+1})$; (b) Os mesmos da alínea anterior.

24. (a) $\dfrac{\partial z}{\partial x}(1,1) = 1/2\,;\ \dfrac{\partial z}{\partial y}(1,1) = 1/2\,;\ \dfrac{\partial^2 z}{\partial y\,\partial x}(1,1) = -3/4\,.$ (b) – (c) –

25. – 26. (a) –; (b) $\dfrac{\partial z}{\partial x}(3,-3) = 1/5\,;\ \dfrac{\partial z}{\partial y}(3,-3) = -4/5\,;\ \dfrac{\partial^2 z}{\partial x\,\partial y}(3,-3) = 18/125\,.$

27. (a) – (b) $\dfrac{\partial x}{\partial y}(1/2,2/e)=-2\,;\ \dfrac{\partial x}{\partial z}(1/2,2/e)=e/4\,;\ \dfrac{\partial^2 x}{\partial z\,\partial y}(1/2,2/e))=-e/4\,.$

28. (a) – (b) $\dfrac{\partial z}{\partial x}(1,0)=\dfrac{\partial z}{\partial y}(1,0)=0$, logo $(1,0)$ é ponto crítico de $z(x,y)$.

29. (a) – (b) $h(2,1)=1$ é mínimo local. 30. (a) – (b) $h_{uv}(0,1)=-30$. 31. (a) – (b) $g'(0)=9$.

32. (a) – (b) $\dfrac{\partial x}{\partial u}(0,1)=2\,;\ \dfrac{\partial x}{\partial v}(0,1)=-1\,;\ \dfrac{\partial^2 x}{\partial v\,\partial u}(0,1)=20\,.$

33. (a) – (b) $f'(0)=0$. (c) $\dfrac{\partial h}{\partial t}(1,1)=-1$. 34. (a) – (b) $f'(0)=0$.

35. $J_g(1,0,1)=[J_f(1,0,0)]^{-1}=\mathrm{diag}\{1/2\,,\,1\,,\,1\}$.

36. (a) $P_0=(x_0,y_0)\,:\,x_0\neq\pi/2+k\,\pi\,,k\in\mathbb{Z}\ \wedge\ y_0\neq 0\,;$ (b) $P_0=(u_0,v_0,w_0)\,:\,w_0\neq k\,\pi\,,k\in\mathbb{Z}\,.$

37. (a) h é localmente invertível numa vizinhança de cada ponto da forma $P_0=(x_0,y_0)\,:\,y_0\neq 0\,;$
(b) $h^{-1}(u,v)=(v/2-\sqrt{v^2-4u}/2,\sqrt{v^2-4u}/2)\,,\ (u,v)\in\mathcal{B}((8,6),R)\,,R>0\,.$

38. (a) – (b) $f^{-1}(u,v)=(\sqrt[3]{u},v)\,,\ (u,v)\in\mathbb{R}^2$. 39. –

40. (a) $J_f(x,y)=\begin{bmatrix}(2y^2-2x^2)/(x^2+y^2)^2 & -4xy/(x^2+y^2)^2\\ 4xy/(x^2+y^2)^2 & (2y^2-2x^2)/(x^2+y^2)^2\end{bmatrix}$; $\det J_f(x,y)=4/(x^2+y^2)^2\neq 0$, $(x,y)\neq(0,0)$. (b) $f^{-1}(u,v)=(2u/(u^2+v^2),-2v/(u^2+v^2))\,,\ (u,v)\neq(0,0)\,.$

41. (a) – (b) $\det J_{f^{-1}}(f(1,2))=e^{-2}$.

42. (a) – (b) $J_\varphi(0,0)=\begin{bmatrix}0&1\\-1&0\end{bmatrix}$. (c) $J_{g\circ\varphi}(0,0)=\begin{bmatrix}-1&0\\0&0\end{bmatrix}$. (d) $\det J_{\phi^{-1}}(0,0)=1$.

43. (a) – (b) $J_g(1,2)=\begin{bmatrix}-3&4\\-2&3\end{bmatrix}$. (c) g é de classe C^1 numa vizinhança de $(1,2)$ e $\det J_g(1,2)=-1\neq 0$ logo g é localmente invertível e $\det J_{g^{-1}}(1,-2)=-1$.

44. (a) F é de classe C^∞ em $\mathbb{R}^2$ e $\det J_F(0,0)=-1\neq 0$ logo F é localmente invertível e $J_{F^{-1}}(1,-1)=\begin{bmatrix}0&1\\1&0\end{bmatrix}$. (b) (i) $D_G=\{(u,v)\in\mathbb{R}^2:u\neq 0\ \wedge\ v\neq 0\}$; (ii) $J_{G\circ F}(0,0)=\begin{bmatrix}-1&1\\1&1\end{bmatrix}$; $J_{F^{-1}\circ G}(1,-1)=\begin{bmatrix}1&1\\1&-1\end{bmatrix}$. 45. –. 46. $\det J_f(x,y)=0\,,\ (x,y)\in\mathbb{R}^2$ logo f_1 e f_2 são funcionalmente dependentes em $\mathbb{R}^2$; $f(\mathbb{R}^2)=\{(u,v)\in\mathbb{R}^2:v=1/u^2\ \wedge\ u>0\}$. 47. $\lambda=4\,,\ \wedge\,\nu=-3$.

48. $\lambda=3$. 49. $A=D_f=\{(x,y)\in\mathbb{R}^2:y\neq 0\ \wedge\ y\neq -x\}$; $f(A)=\{(u,v)\in\mathbb{R}^2:v=(u-1)/(u+1)\}$.

Algumas noções geométricas do cálculo diferencial: curvas e superfícies.

50. – 51. (a) $x=4\,\arccos y/2\,,\ z=3/2\sqrt{4-y^2}\,,\ y\in[-2,2]$; (b) $y=2\,\cos x/4\,,\ y^2/4+z^2/9=1\,,$ $x\in[0,8\pi]$; (c) $x+1=3\,y^2\,,\ y>0$. (d) $x=y\,,\ y^2/2+z^2/4=1\,.$

52. (a) $x = t\,,\ y = t\,,\ z = 2\,t^2\,,\ t \in \mathbb{R}\,;$ (c) $x = \cos t\,,\ y = \sin t\,,\ z = 1\,,\ t \in [0, 2\pi]\,;$ (b) $x = 1/\sqrt{5}\cosh t\,,$ $y = 2/\sqrt{5}\cosh t\,,\ z = \sinh t \ \vee\ x = -1/\sqrt{5}\cosh\, t\,,\ y = -2/\sqrt{5}\cosh\, t\,,\ z = \sinh t\,,\ t \in \mathbb{R}\,.$

53. (a) $x = -1\,,\quad y = 2\,\pi - 2z\,;$ (b) $3x + 2z = 1\,,\quad y = 2\,.$

54. Recta tangente a Γ_1 em $(\pi/4, \sqrt{2}/2, \sqrt{2}/2):\ x - \pi/4 = 1 - \sqrt{2}\,y\,,\quad z + y = \sqrt{2}\,;$
Plano normal a Γ_1 em $(\pi/4, \sqrt{2}/2, \sqrt{2}/2):\ x - \sqrt{2}/2\,y + \sqrt{2}/2\,z - \pi/4 = 0\,.$
Recta tangente a Γ_2 em $(1,1):\ 5x - 4y = 1$; Recta normal a Γ_2 em $(1,1):\ 5y + 4x = 9\,.$

55. Recta tangente: $(x+2)/54 = (y-1)/56 = (z-6)/8$; Plano normal: $27x + 28y + 4z + 2 = 0\,.$

56. (a) Plano que passa em $(1,0,2)$ e é perpendicular ao vector $(17,10,-6)$; uma equação cartesiana deste plano é $17x + 10y - 6z = 5\,.$ (b) Parabolóide elíptico de equação cartesiana $z = x^2 + y^2\,.$ (c) Cone elíptico de equação cartesiana $x^2 = y^2 + z^2\,.$ (d) Cilindro circular de equação cartesiana $y^2 + z^2 = 1\,.$

57. $r(s,t) = (1+s+t, 2+s-t, -3+s+t)\,,\ (s,t) \in \mathbb{R}^2\,.$ 58. $r(s,t) = (s, t, \pm\sqrt{1+s^2+t^2})\,,\ (s,t) \in [-1,1] \times [-3,3]\,.$ 59. (a) Plano tangente: $\sqrt{2}x + \sqrt{2}y + z = 3$; Recta normal: $(x - \sqrt{2}2)/\sqrt{2} = (y - \sqrt{2}/2)/\sqrt{2} = z - 1\,.$ (b) – 60. (a) Plano tangente: $2x + 2y - z = 2$; Recta normal: $(x-1)/(-2) = (y-1)/(-2) = z - 2$; (b) – 61. Plano tangente: $2z = y$; Recta normal: $x = 0\,,\quad 2y + z = 5\,.$ 62. (a) $2x - 4y - z = 5$; (b) $x + y + z = 2$; (c) $\cos(\theta)\,x - \sin(\theta)\,y = r$; (d) $3x + 4y = 25\,.$

63. Se $S_1 : x^2/16 + y^2/9 = z^2\,,\ S_2 : x^2 + y^2 + (z-10)^2 = 90\,,\ P_0 = (0,3,1)\,,\ P_1 = (0,-3,1)$, então
Plano tangente a S_1 em $P_0:\ y = 3z$; Plano tangente a S_2 em $P_0:\ y = 3z$;
Plano tangente a S_1 em $P_1:\ y = -3z$; Plano tangente a S_2 em $P_1:\ y = -3z\,.$

64. – 65. $(2,0,c)\,, (0,0,c)\,, (1,1,c)\,, (1,-1,c)\,,\ c \in \mathbb{R}\,.$ 68. (a) – ; (b) –

66. $x + y + z = 11\,\sqrt{6}/6\,,\ x + y + z = -11\,\sqrt{6}/6\,;$ 67. $(4/3, 4/3, 1/3)\,,\ (-4/3, -4/3, -1/3)\,.$

69. (a) $\dfrac{\partial F}{\partial y}(x,y,z) = G_u(xy, x+y, z^2)\,x + G_v(xy, x+y, z^2)\,;\ \dfrac{\partial F}{\partial z}(x,y,z) = 2z\,G_w(xy, x+y, z^2)\,;$ $\dfrac{\partial^2 F}{\partial x^2}(x,y,z) = y^2\,G_{u^2}(xy, x+y, z^2) + 2y\,G_{uv}(xy, x+y, z^2) + G_{v^2}(xy, x+y, z^2)\,.$ (b) $x + z = 0\,.$

70. (a) – (b) $\dfrac{\partial f}{\partial x}(1,0) = -\sqrt{2}/2\,,\ \dfrac{\partial f}{\partial y}(1,0) = -2\,\sqrt{2}\,.$ (c) $x + 4y + \sqrt{2}\,z = 2\,.$ 71. (a) –; (b) –.

2. Cálculo Integral

Superfícies quádricas.

72. (a) Em $\mathbb{R}^2$, o conjunto definido pela equação $z = y^2$ é uma parábola no plano YOZ com vértice $(0,0)$, eixo ZZ' e concavidade voltada para a parte positiva de ZZ'.
(b) Em $\mathbb{R}^3$, o conjunto definido pela equação $z = y^2$ é um cilindro parabólico; a directriz é a parábola de equação $z = y^2$, no plano YOZ e as geratrizes são paralelas a XX'.

73. (a) Relativamente à superfície $x^2+2\,y^2+z^2=1$: — o traço no plano $x=k$, $k\in[-1,1]$, é o ponto $(-1,0,0)$ se $k=-1$; o ponto $(1,0,0)$ se $k=1$; a elipse de equação $x^2/(\sqrt{1-k^2/2})^2+z^2/(\sqrt{1-k^2})^2=1$, se $k\in]-1,1[$; — o traço no plano $y=k$, $k\in[-1/\sqrt{2},1/\sqrt{2}]$, é o ponto $(0,-1/\sqrt{2},0)$ se $k=-1/\sqrt{2}$; o ponto $(0,1/\sqrt{2},0)$ se $k=1/\sqrt{2}$; a circunferência de equação $x^2+z^2=(\sqrt{1-2k^2})^2$, se $k\in]-1/\sqrt{2},1/\sqrt{2}[$; — o traço no plano $z=k$, $k\in[-1,1]$, é o ponto $(0,0,-1)$ se $k=-1$; o ponto $(0,0,1)$ se $k=1$; a elipse de equação $x^2/(\sqrt{1-k^2})^2+y^2/(\sqrt{1-k^2/2})^2=1$, se $k\in]-1,1[$; Relativamente à superfície $y^2=x^2+z^2$: — o traço no plano $x=k$, $k\in\mathbb{R}$, é o par de rectas $y=z\ \vee\ y=-z$ se $k=0$; a hipérbole equilátera de equação $y^2/k^2-z^2/k^2=1$, se $k\neq 0$; o traço no plano $y=k$, $k\in\mathbb{R}$, é o ponto $(0,0,0)$ se $k=0$; a circunferência de equação $x^2+z^2=k^2$, se $k\neq 0$; o traço no plano $z=k$, $k\in\mathbb{R}$, é o par de rectas $y=x\ \vee\ y=-x$ se $k=0$; a hipérbole equilátera de equação $y^2/k^2-x^2/k^2=1$, se $k\neq 0$; (b) A equação $x^2+2\,y^2+z^2=1$ representa um elipsóide com centro $(0,0,0)$ e de semi-eixos $a=1$, $b=1/\sqrt{2}$ e $c=1$. A equação $y^2=x^2+z^2$ representa uma superfície cónica de vértice $(0,0,0)$ e eixo YY'.

74. (a) Cilindro circular; a directriz é a circunferência de equação $x^2+z^2=9$ e as geratrizes são paralelas a YY'; (b) Hiperbolóide de 1 folha com eixo YY'; (c) Elipsóide com centro $(0,0,0)$; (d) Hiperbolóide de 2 folhas com eixo ZZ'; (e) Cone circular com vértice $(0,0,4)$ e eixo ZZ'; (f) Parabolóide elíptico com vértice $(0,0,0)$ e eixo YY' e concavidade voltada para a parte positiva de YY'; (g) Esfera com centro $(0,0,-1)$ e raio 1; (h) Parabolóide hiperbólico com eixo YY'; (i) Parabolóide elíptico com vértice em $(0,0,4)$, eixo ZZ' e concavidade voltada para a parte negativa de ZZ'; (j) Cilindro parabólico; a directriz é a parábola de equação $z=2+y^2$ e as geratrizes são paralelas a XX'.

75. (a) $S_1:(x+3/2)^2/(\sqrt{13}/2)^2+y^2/(\sqrt{13}/2\sqrt{2})^2-z^2/(\sqrt{13}/2)^2$, é um hiperbolóide de uma folha com centro $(-3/2,0,0)$ e eixo ZZ'. $S_2:x^2/(5(4\sqrt{2}))^2+(y-5/8)^2/(5/8)^2-z^2/(5/(4\sqrt{2}))^2$, é um hiperbolóide de 1 folha com centro $(0,5/8,0)$ e eixo ZZ'; (b) $S_1\cap S_2=\{(x,y,z)\in\mathbb{R}^3: x^2+2\,y^2-z^2+3x=1\ \wedge\ 6x+5y=2\}$, logo está contida no plano de equação $6x+5y=2$.

76. (a) S_1 é um parabolóide com vértice $(0,0,0)$, eixo ZZ' e concavidade voltada para a parte positiva de ZZ'. S_2 é um cilindro parabólico com geratrizes paralelas a XX'; (b) $\mathcal{C}=\{(x,y,z)\in\mathbb{R}^3: x^2+2\,y^2=1\wedge z=1-y^2\}$. A projecção de $\mathcal{C}$: — no plano XOY é a elipse de equação $x^2+2\,y^2=1$; — no plano YOZ é a parábola de equação $z=1-y^2$; — no plano XOZ é a parábola de equação $x^2=2\,(z-1/2)$. 77. –; 78 –.

Integral duplo. Definição e propriedeades.

79. (a) 60; (b) $75/2$; (c) 3; (d) 4π. 80. –. 81. (a) >0; (b) >0; (c) >0; (d) 0; (e) 0; (f) 0; (g) <0; (h) 0; (i) <0; (j) 0; (k) 0; (l) >0; (m) >0; (n) >0; (o) 0; (p) 0.

Cálculo do integral duplo em coordenadas cartesianas.

82. (a) $9/20$; (b) $-9/8$; (c) $-17/20$. 83. (a) $\int_0^1 \int_{\arcsin y}^{\pi/2} f(x,y)\,dx\,dy$; (b) $\int_0^2 \int_0^{2x} f(x,y)\,dy\,dx$; (c) $\int_0^1 \int_{e^y}^2 f(x,y)\,dx\,dy$; (d) $\int_0^1 \int_0^{\sqrt{x}} f(x,y)\,dy\,dx + \int_1^2 \int_0^{2-x} f(x,y)\,dy\,dx$; (e) $\int_0^{\pi/4} \int_0^{\tan y} f(x,y)\,dx\,dy$; (f) $\int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{4-2y^2}}^{\sqrt{4-2y^2}} f(x,y)\,dx\,dy$; (g) $\int_0^2 \int_{2-\sqrt{4-y^2}}^{y} f(x,y)\,dx\,dy$; (h) $\int_0^1 \int_{\sqrt{y}}^{3-2y} f(x,y)\,dx\,dy$. 84. (a) $\int_0^1 \int_0^v \sqrt{1-v^2}\,du\,dv = 1/3$; (b) $\int_0^{\pi/2} \int_0^{\sin x} \cos x\,\sqrt{1+\cos^2 x}\,dy\,dx = 2\sqrt{2} - 1/3$; (c) $\int_0^9 \int_0^{\sqrt{x}} y\,\cos(x^2)\,dy\,dx = 1/(4\sin 81)$; (d) $\int_0^1 \int_0^{x^2} \sqrt{x^3+1}\,dy\,dx = 2/9\,(2\sqrt{2}-1)$; (e) $\int_0^2 \int_0^{y/2} e^{y^2}\,dx\,dy = 1/4\,(e^4-1)$. 85. (a) $-1/4$; (b) $7/6 \ln 2$; (c) $1/6$; (d) $8/3 \ln 2$; (e) $e^4 - 4\,e/2$; (f) $1/8$; (g) $8/3$; (h) $6\sqrt{6} - 8\sqrt{3}$.

Mudança de variável no integral duplo.

86. (a) $19\,(2-\sqrt{2})/6$; (b) 3π; (c) $32/5\,\pi$; (d) $\pi/2$; (e) $7/6\,\pi - \sqrt{3}/4$. 87. (a) –; (b) $\ln\sqrt{2} + 1/(2\sqrt{2}) - 1/2$; (c) $I = \int_{1/2}^1 \int_0^{\pi/6} \sin 2\theta/\sqrt{\cos 2\theta}\,d\theta\,dr + \int_1^{\sqrt{2}} \int_{1/2 \arccos 1/(r^2}^{\pi/6} \sin 2\theta/\sqrt{\cos 2\theta}\,d\theta\,dr$. 88. (a) $2\sinh 1$; (b) $1/3\,\pi^4$; (c) $3/2$. 89. –.

Algumas aplicações do integral duplo: Áreas de regiões planas.

90. (a) $64/3$; (b) $e^2 + e^{-2} - 2$; (c) $\pi/12 + \sqrt{3}/4 - 1/2$; (d) $1/(2\sqrt{2}) \arctan 1/\sqrt{2}$; (e) $3 \arctan 2/3 - \arctan 2$; (f) 9; (g) $3 - \ln 4 - 9/8$. 91. $\pi/3 - \sqrt{3}/4$. 92. (b) –;
92. (a) $I = \int_0^{\pi/4} \int_0^{\sin\theta} r\,dr\,d\theta + \int_{\pi/4}^{\pi/2} \int_0^{\cos\theta} r\,dr\,d\theta$; (c) $\pi/8 - 1/4$. 93. (a) –; (b) (iii), (iv), $(viii)$; (c) $5/3\,\pi - 2\sqrt{3}$. 94. (a) $R = \{(x,y) \in \mathbb{R}^2 : 0 \le x \le 3 \ \wedge\ x \le y \le 2x\}$; (b) $9/2$.

Volumes.

95. (a) $2\sqrt{2}/3$; (b) π; (c) 22π; (d) $48\pi - 64/9$; (e) $6\sqrt{3} - 5/3\,\pi$; (f) 1; (g) $32/3\,\pi$; (h) $3/16\,\pi$. 96. $4/3\,\pi$.

Massa, Momentos e Centro de massa.

97. (a) $m = a^4 k/6$, $M_x = M_y = a^5 k/15$, $C = (2/5\,a, 2/5\,a)$; (b) $m = 13/20$, $M_x = 19/42$, $M_y = 3/10$, $C = (6/13, 190/273)$; (c) $m = a^4/2$, $M_x = a^4/2$, $M_y = 0$, $C = (0, 3\,a/2\,\pi)$; (d) $m = 12$, $M_x = 18$, $M_y = 24$, $C = (2, 3/2)$.

Integral triplo. Cálculo do integral triplo em coordenadas cartesianas

98. (a) $\ln \sqrt{2} - 5/16$; (b) 4; (c) 0. 99. (a) (i) $\displaystyle\int_0^{2\sqrt{2}} \int_0^{\sqrt{8-y^2}/2} \int_{(y^2+4z^2)/4}^{2} x\,dx\,dz\,dy$; (ii) $\displaystyle\int_0^{\sqrt{2}} \int_{z^2}^{2} \int_0^{\sqrt{4x-4z^2}} x\,dy\,dx\,dz$; (b) $4/3\,\pi$.

Mudança de variável

100. (a) 4π; (b) $4\pi \ln 3/2$; (c) $3/10\,\pi$; (d) 0; (e) $7/6\,\pi$; (f) $3/2\,\pi^2$; (g) 0. 101. (a) 0; (b) $-$; (c) $\displaystyle\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{x^2+y^2-1}^{2-x} 4\,y\,dz\,dy\,dx$. 102. $\displaystyle\int_{-\sqrt{3}/2}^{\sqrt{3}/2} \int_0^{\sqrt{3/4-x^2}} \int_{\sqrt{x^2+y^2/3}}^{\sqrt{1-x^2-y^2}} \sqrt{x^2+y^2+z^2}\,dz\,dy\,dx$.

Aplicações do integral triplo: Volumes.

103. (a) $4/3\,\pi\,r^3$; (b) 2π; (c) $\pi/3$; (d) $\pi/6$; (e) $64/3\,\pi\,(\sqrt{2}-1)$; (f) $10/3\,\pi$. 104. (a) $14/3\,\pi$; (b) $(8\sqrt{3}-9)/8\,\pi$.

Massa, Momentos e Centro de massa.

105. $124/5\,k\,\pi$. 106. $C = (0,0,1/2)$, $M_{xy} = 1/12\,k\,\pi$. 107. $C = (0,0,9/4)$. 108. $m = \pi/2 \arctan 1/2$. 109. $m = 11/4\,k\,\pi$.

Integral curvilíneo. Integral curvilíneo de funções escalares.

110. (a) $1/3\,(\sqrt{(1+e^2)^3} - 2\sqrt{2})$; (b) $\sqrt{5}\ln 2$; (c) $-16 - 33/8\sqrt{17} + 1/32\ln(\sqrt{17}+4)$; (d) 12π. 111. (a) $2\sqrt{2}\,\pi^2$; (b) $2\sqrt{5}/3$; (c) 45π; (d) $2/3\,\pi$. 112. $-$. 113. (a) $x = 1/2 + 1/2\cos t$, $y = 1/\sqrt{2}\sin t$, $z = 1/2 - 1/2\cos t$, $t \in [0, 2\pi]$; (b) $\sqrt{2}\,\pi$. 114. (a) $\pi/2$; (b) 2. 115. 360 euros. 116. $m = 72$; $C = (0,0)$.

Integral curvilíneo de funções vectoriais: Cálculo e aplicações.

117. (a) $3\pi - 8/3$; (b) 3; (c) 2; (d) π. 118. (a) $-\pi/4 - 2/\pi$; (b) -4π. 119. $1/2$. 120. 0.

Campos conservativos. Independência do caminho.

121. $-2\sin 2$. 121. (a) $g(x,y,z) = xyz$; (b) $1/2\,e^4\sin 2$. 122. $64/3$. 123. $\phi(x) = 2x - 4$. 124. -30.

Teorema de Green.

126. 8π. 127. $64/15$. 128. $A(R) = -k$. 129. $1 + \pi/8$. 130. (a) $k = 1/6$; (b) $-$. 131. (a) $56/15$; (b) $28/15$. 132. $a = 3$. 133. $\pi/16$. 134. (a) 2π; (b) 0. 135. $\displaystyle\int_C (-y\,dx + x\,dy)/(x^2+y^2) = 0$ se $r < \sqrt{2}$ e é igual a 2π se $r > \sqrt{2}$. 136. (a) $2\arctan 2$; (b) 2π.

Integral de superfície. Integral de superfície de campos escalares.

137. (a) 0; (b) >0; (c) <0; (d) <0; (e) 0; (f) 0; (g) 0; (h) 0; (i) 0; (j) >0; (k) 0; (l) 0; (m) <0; (n) <0. 138. (a) $40\sqrt{2}\,\pi$; (b) 0; (c) $16\,\pi$; (d) $128/3\,\pi$; (e) $40\,\pi$; (f) $5/12\,\sqrt{5}\,\pi + 31/60\,\pi$; (g) $32/3$. 139. (a) $24\,\pi$; (b) $24\,\pi$; (c) $60\,\pi$. 140. 0.

Aplicações.

141. (a) $4\,\pi\,a^2$; (b) $(17\sqrt{17}+47)/6\,\pi$; (c) $36\,(\sqrt{2}+1)\,\pi$. 142. (a) $18\,\pi$; (b) $\sqrt{6}\,\pi$; (c) $\pi/4 - \arctan 2/2$. 143. (a) $S_1 : r_1(s,t) = (s\,\cos\,t, -1+\sqrt{2}\,s\,\sin\,t, 2-\sqrt{2}\,s\,\sin\,t)$, $0 \le t \le 2\,\pi, 0 \le s \le 1$; $S_2 : r_2(x,y) = (x, y, \sqrt{2x^2+2y^2}), (x,y) : x^2 + (y+1)^2/2 \le 1$; (b) $A(S_1) = 2\,\pi$; $A(S_2) = \sqrt{6}\,\pi$. 144. (a) $8\,\pi^2$; (b) $22\,\pi^2$.

Integral de superfície de campos vectoriais: Definição, propriedades e cálculo.

145. (a) (i) <0; (ii) >0; (iii) 0; (iv) 0; (v) 0; (b) (i) 0; (ii) 0; (iii) 0; (iv) <0; (v) 0. 146. (a) $72\,\pi$; (b) 0; (c) 0; (d) $36\,\pi$; (e) $4\,\pi/e$; (f) $(8-5\sqrt{2})/12\,\pi$; (g) $10/3\,\pi$. 147. $2\,\pi$. 148. $0\,m^3/s$. 149. (a) $4\,\pi\,q$; (b) –; (c) $4\,\pi\,q$.

3. Teoremas Fundamentais do Cálculo Vectorial

Teorema de Stokes e Teorema da Divergência.

150. –. 151. –. 152. (a) div $E = 3 - p/\|\,r\,\|^p$. $p = 3$. 153. –. 154. (a) $81/8$; (b) (i) $\pi/2$; (ii) $\pi/8$. 155. (a) $64\,\pi$; (b) $144\,\pi$. 156. (a) $3/2\,\pi$; (b) $9/4\,\pi$. 157. (a) $8\,\pi$; (b) $4\,\pi$. 158. $128\,\pi$. 159. 0. 160. (a) $7\,\pi$; (b) $63\,\pi$. 161. (a) $7/3\,\pi$; (b) 0. 162. (a) 0; (b) $4\,\pi$. 163. (a) –; (b) (i) –; (ii) $4\,\pi$. 164. –. 165. –. 166. (a) $-2\,\pi$; (b) 0; (c) $4/3$. 167. –. 168. (a) $-\pi/2$; (b) $-\pi/2$. 169. (a) –; (b) $75\,\pi$. 170. (a) π; (b) 0; (c) π; (d) $F(x,y,z)$; (e) π. 171. $3/2\,\pi$. 172. $-42\,\pi$. 173. (a) $2/3$; (b) 0;. 174. $9\,\pi$. 175. 16. 176. $-75\,\pi$. 177. $3/2\,\pi - 1/2$. 178. (a) $a = -1/3$, $b = 0$, $c = 1$; (b) i. $G(x,y,z) = (0, xz^3, -1/3x^3y)$; ii. $96\,\pi$.