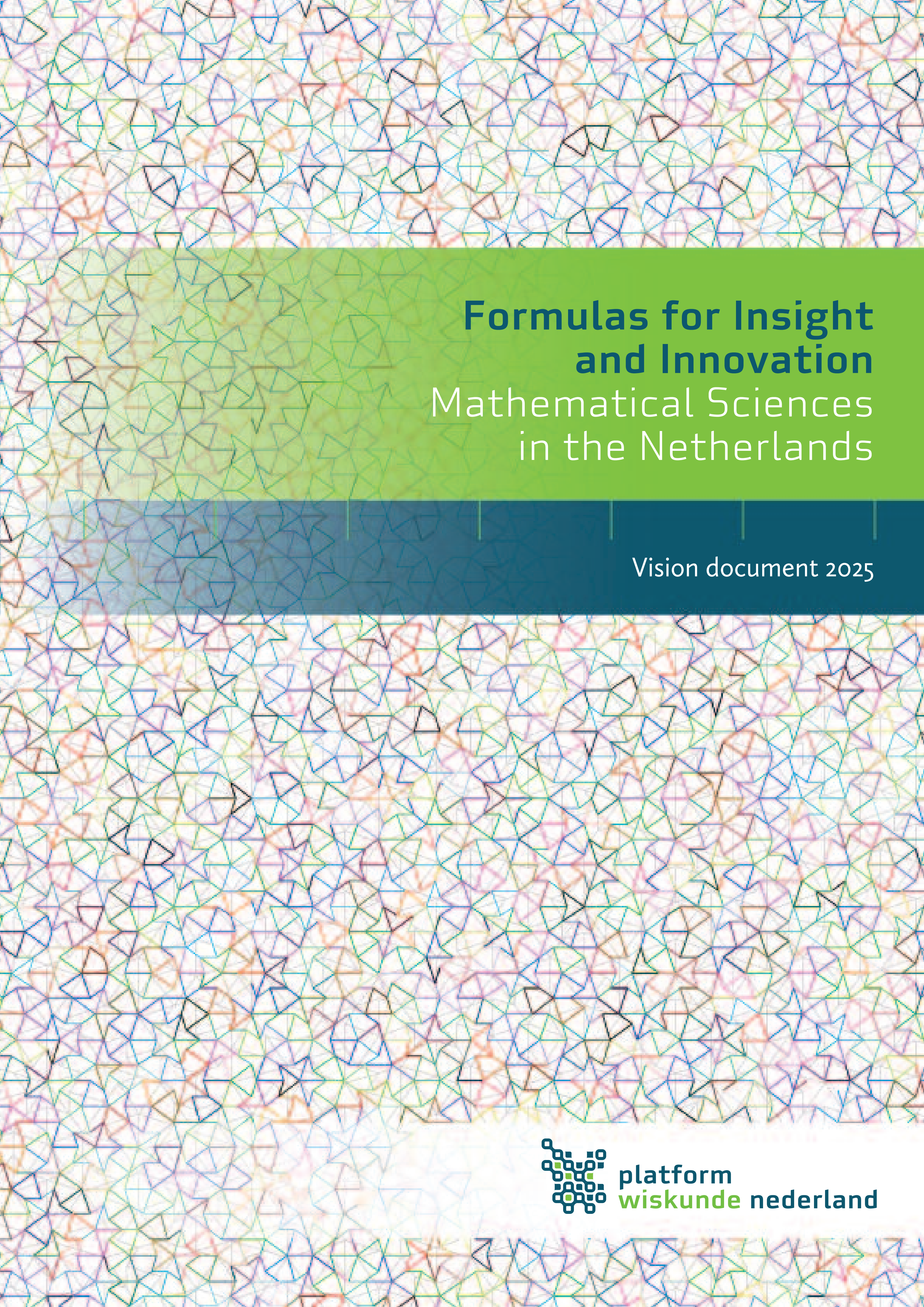
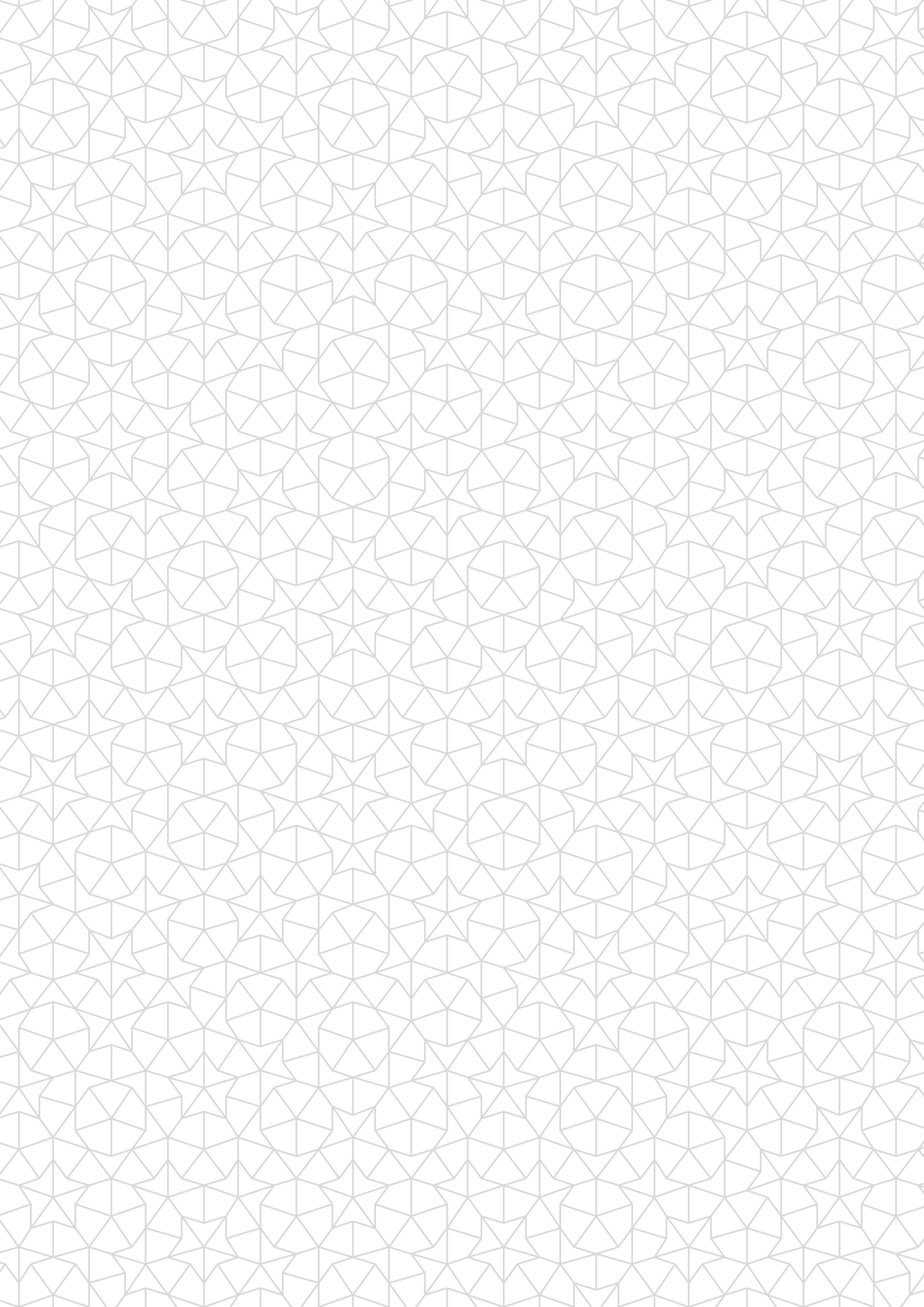


# Formulas for Insight and Innovation

## Mathematical Sciences in the Netherlands

Vision document 2025

platform
wiskunde nederland



*Film is one of the three universal languages.*
*The other two: mathematics and music.*

*– Frank Capra –*

# Formulas for Insight and Innovation

## Mathematical Sciences in the Netherlands

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# Preface

What will our world look like in 2025? And what will be the role of the mathematical sciences in shaping that world?
Since the start of the 21st century it has become clear that the mathematical sciences are gaining a new stature. They are increasingly providing the knowledge to enable innovation breakthroughs and insights in many other disciplines such as biology, healthcare, social sciences and climatology, alongside their traditional role in physics, chemistry and computer science. The importance of the mathematical sciences is also rapidly increasing in the business world, for example in design processes, electronics and finance.

All these developments are vital for economic growth and competitive strength, and demand an in-depth review of the overall way we look at the mathematical sciences. This involves the integration of mathematics with statistics, operations research and computational science, it carries implications for the nature and scale of research funding, and it motivates a rethinking of curricula in a broad range of educational programs in which the mathematical sciences play an ever-increasing role.

Platform Wiskunde Nederland (PWN) wishes to gain a good understanding of the role of the mathematical sciences in society and of today's needs, taking into account present trends and the internal dynamics of the field. It has asked Jan Karel Lenstra, former dean of Mathematics and Computer Science at Eindhoven University of Technology and former director of Centrum Wiskunde & Informatica in Amsterdam, together with the mathematics community in the Netherlands, to formulate a vision for the medium term. I would like to express my gratitude towards the writing committee and to all those who have committed time by reviewing the drafts or by sharing their views in consultations.

The vision document contains many important observations and recommendations. It should send a strong signal to the government, NWO, academia and other stakeholders, including the professionals who are involved in education, research or applications of the mathematical sciences.

We have to recognize that to a large extent the mathematics community itself needs to take charge of its future role in science and society. PWN, the primary point of contact in the Netherlands for everything relating to the mathematical sciences, intends to make every effort in the coming years to help put the recommendations into practice.

C.J. (Hans) van Duijn
Chairman, Platform Wiskunde Nederland

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# Summary

This vision document was commissioned by Platform Wiskunde Nederland (PWN). Its purpose is to present the vision and ambition of the mathematical sciences in the Netherlands in the medium term, say, to 2025, and to sketch the road ahead for education, research and innovation.

The document addresses a broad audience, including the Ministry of Education, Culture and Science, the Ministry of Economic Affairs, the Netherlands Organisation for Scientific Research, deans of science faculties, directors of mathematics departments, as well as the mathematical scientists themselves. The document describes major trends in society and science that influence the mathematical sciences, and presents a selection of grand mathematical challenges. It analyzes the mathematical sciences in the Netherlands in terms of their strengths, weaknesses, opportunities and threats, and concludes with a number of recommendations.

In 2025, the world will be an intelligent global society. The role of the mathematical sciences is to provide the abstractions, concepts, methods and techniques that will enable the shaping of that world.
Over the past centuries, mathematics has grown to be the indispensable but largely invisible foundation supporting science, technology, and much of society itself. Many are unaware that the functioning of their world depends on mathematical insights and tools, and that the evolution of that world can only occur because mathematics is always innovating. A recent report by Deloitte commissioned by PWN indicates that the mathematical sciences create thirty percent of Dutch national income and that a strong mathematical foundation is critical to the success of an advanced economy. In the 21st century, the role of the mathematical sciences will become even more important, with advances in computing and the availability of data as major drivers.

The increased reach of the mathematical sciences has given a new stature to the field. This has ramifications for research funding and education. It is crucial for the health of the mathematical sciences to maintain and develop a strong core, which is the primary source for innovation in the long run. It is also crucial to rethink mathematics education, including the teaching of mathematics to students in other disciplines.

In the present document, we observe six major societal and scientific trends that influence the mathematical sciences:

- *globalization*, which has far-reaching effects in science and education;
- the proliferation of *computation*, which has transformed both the mathematical sciences and its application areas;
- *big data*, which, next to computation, is a major driver of the increasing reach of the mathematical sciences;
- *math inside*, which refers to the much-intensified interplay between mathematics and its broadening array of applications;
- the growing *coherence* of the mathematical sciences, with breakthroughs occurring across subdisciplinary borders;
- the mathematicians’ awareness of the need to be *accountable* for their work to society at large.

We formulate five grand challenges for the mathematical sciences: the Langlands program, complex systems, stochastic modeling, efficient computation, and data science.

We carry out a SWOT analysis for the situation in the Netherlands on three dimensions: higher education and research, primary and secondary education, and society and innovation. The SWOT analysis yields nine recommendations that demand attention in the coming decade.

1. *Balance workload.* Expand the mathematics staff in academia by at least fifty percent, in order to balance teaching and research and to safeguard their quality.

2. *Consolidate dynamic clusters.* Provide structural funding to the four mathematics clusters. The clusters should evolve in response to developments of the discipline and initiate national graduate education programs.

3. *Reform academic mathematics teaching.* In view of the increased and more diverse demand for mathematical scientists, academia needs to reassess the mathematics education. Mathematicians should be in charge of teaching mathematics to students in other subjects.

4. *Secondary eduation and academia.* The universities need to resume their role in educating high school teachers, by making the curriculum flexible and attractive. A scheme should be developed for creating and stimulating dual appointments between high schools and universities.

5. *Foster talent.* Primary schools need tracks and tools for fostering excellence. High schools should maintain the *Wiskunde D* elective. The universities should create career paths for attracting and keeping mathematical talent, especially among women and minorities.

6. *Funding in transition.* The budget of NWO for funding fundamental research should be expanded. Mathematicians need to embrace the many opportunities offered by European programs, by emerging areas of the mathematical sciences, by larger cooperative initiatives, and by strategic alliances with industry. PWN should take charge of promising thematic opportunities.

7. *Reaching out.* Mathematicians should expand their outreach activities. PWN should help by professionalizing communication.

8. *Vision of science.* The Dutch government needs to formulate a long-term vision of science, recognizing that fundamental research is the principal source for innovation in the long run. The academic community should actively engage in this process.

9. *Implementation plan.* The government should establish a committee with the task of drafting an implementation plan for the Dutch mathematical sciences in the medium term. The committee will formulate action lines for the mathematical scientists in higher education and research, in primary and secondary education, and in society and innovation. It will set measurable milestones on relevant performance criteria, which will allow the government to monitor the effectiveness of additional funds and to make these funds structural after proven success.

# Samenvatting

Dit visiedocument is geschreven in opdracht van het Platform Wiskunde Nederland (PWN). Het presenteert de visie en ambitie van de Nederlandse wiskunde op de middellange termijn, en laat een licht schijnen over de ontwikkeling van onderwijs, onderzoek en innovatie. We vatten wiskunde hier op in ruime zin: het omvat ook statistiek, operations research, computational science en de toepassing daarvan in andere gebieden.

Het document is geschreven voor beleidsmakers, zoals het Ministerie van Onderwijs, Cultuur en Wetenschap, het Ministerie van Economische Zaken, NWO, decanen van bètafaculteiten en directeuren van wiskunde-instituten, maar ook voor de wiskundigen zelf. Het document beschrijft ontwikkelingen in maatschappij en wetenschap die voor de wiskunde van belang zijn en formuleert enkele grote wiskundige uitdagingen. Het analyseert de wiskunde in Nederland in termen van krachten, zwaktes, kansen en bedreigingen en sluit af met een aantal aanbevelingen.

In 2025 leven we in een intelligente en geglobaliseerde wereld. De wiskunde zorgt voor de abstracties, concepten, methoden en technieken die ons helpen die wereld te vormen.
In de afgelopen eeuwen heeft de wiskunde zich ontwikkeld tot het onmisbare, maar vrijwel onzichtbare, fundament van wetenschap, technologie en een groot deel van de maatschappij. Velen zijn zich er niet van bewust dat hun wereld alleen kan bestaan dankzij wiskundig inzicht en gereedschap, en dat vooruitgang te danken is aan wiskundige innovatie. Een recent rapport van Deloitte, opgesteld in opdracht van PWN, laat zien dat de wiskunde verantwoordelijk is voor dertig procent van het Nederlands bruto nationaal product en dat een stevige wiskundige basis van wezenlijk belang is voor een moderne economie. In de 21e eeuw zal de rol van de wiskunde nog groter worden, vooral door de stijgende kracht van computers en algoritmen en de beschikbaarheid van grote dataverzamelingen.
De toegenomen reikwijdte heeft de wiskunde een nieuw gewicht gegeven. Dit heeft gevolgen voor de financiering van onderzoek en voor het onderwijs. De wiskunde kan alleen gezond blijven als deze zich in de kern kan blijven ontwikkelen, want die kern is de uiteindelijke bron van vernieuwing. Het onderwijs in de wiskunde is aan een kritische heroverweging toe, ook het wiskunde-onderwijs aan studenten in andere vakken.

In dit document schetsen we zes belangrijke maatschappelijke en wetenschappelijke ontwikkelingen die voor de wiskunde van belang zijn:

- de *globalisering*, die verstrekkende gevolgen heeft voor wetenschap en onderwijs;
- de explosieve groei in *rekenkracht*, die voor een transformatie zorgt van zowel de wiskunde als zijn toepassingsgebieden;
- de beschikbaarheid van *big data*, die, naast rekenkracht, de tweede oorzaak is van de toenemende reikwijdte van de wiskunde;
- *math inside*, wat verwijst naar de steeds hechtere interactie tussen de wiskunde en het zich verbredende veld van toepassingen;
- de toenemende *coherentie* binnen de wiskunde zelf, waarbij deelgebieden naar elkaar toegroeien en door-

braken optreden op hun grensvlakken;
- het groeiende besef bij wiskundigen dat ze aan de maatschappij *rekenschap* moeten afleggen voor hun werk.

We formuleren vervolgens vijf grote wiskundige uitdagingen: het Langlands-programma, complexe systemen, stochastische modellering, efficiënte berekeningen, en data science.

We voeren daarna een SWOT-analyse uit voor de Nederlandse wiskunde, waarbij we een onderscheid maken tussen hoger onderwijs en onderzoek, basis- en voortgezet onderwijs, en maatschappij en innovatie. Deze analyse leidt tot negen aanbevelingen die in het komende decennium om aandacht vragen.

1. *Breng de werklast in evenwicht.* Breid de universitaire wiskundestaf uit met ten minste vijftig procent, om de balans tussen onderwijs en onderzoek te herstellen en de kwaliteit van beide te waarborgen.

2. *Consolideer dynamische clusters.* Voorzie de vier wiskundeclusters van structurele financiering. De clusters moeten zich blijven aanpassen aan ontwikkelingen binnen het vakgebied en nationale programma's voor onderwijs aan promovendi starten.

3. *Hervorm het universitaire wiskunde-onderwijs.* Gezien de gestegen en meer uiteenlopende vraag naar wiskundigen moeten de universiteiten hun wiskunde-onderwijs heroverwegen en vernieuwen. Het wiskunde-onderwijs aan studenten in andere vakken moet worden verzorgd door wiskundigen.

4. *Voortgezet en hoger onderwijs.* De universiteiten moeten hun lerarenopleiding flexibel en aantrekkelijk maken en daarmee hun rol in het opleiden van leraren hernemen. Er moet een plan worden opgesteld voor het stimuleren van duale aanstellingen in voortgezet en hoger onderwijs.

5. *Koester talent.* Basisscholen hebben programma's en hulpmiddelen nodig om getalenteerde leerlingen te stimuleren. Middelbare scholen moeten het keuzevak Wiskunde D blijven aanbieden. De universiteiten moeten loopbaanpaden creëren om wiskundig talent aan te trekken en te behouden, in het bijzonder voor vrouwen en minderheden.

6. *Financieringsbronnen.* Het budget van NWO voor het financieren van fundamenteel onderzoek moet worden verhoogd. Wiskundigen moeten daarnaast nieuwe bronnen gaan aanboren: Europese programma's, opkomende toepassingsgebieden, grotere samenwerkingsverbanden, strategische allianties met het bedrijfsleven. PWN moet kansrijke onderwerpen in kaart gaan brengen.

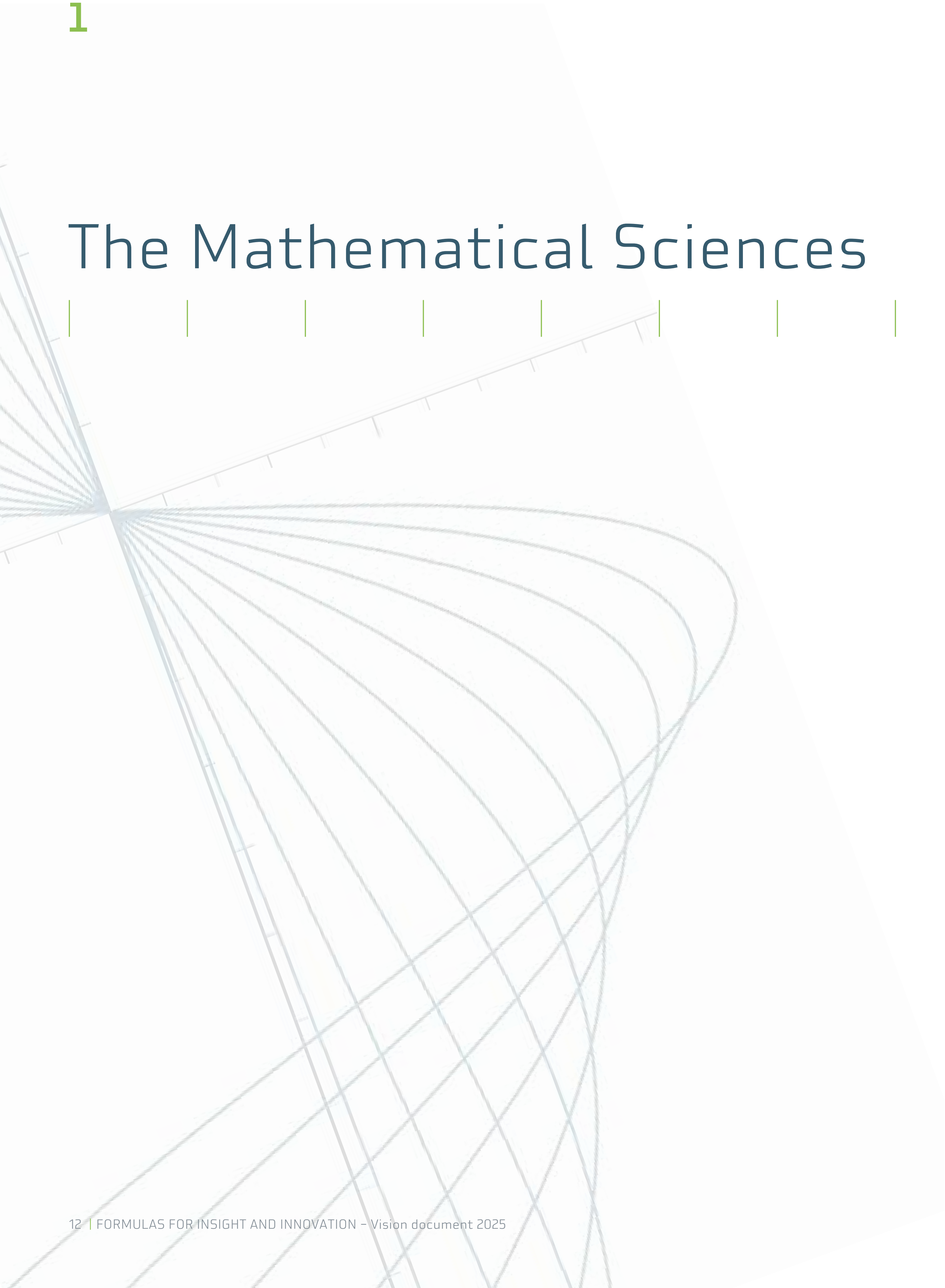
7. *Communicatie.* Wiskundigen moeten, meer nog dan nu, het brede publiek inzicht geven in hun werk. PWN kan helpen door de communicatie te professionaliseren.

8. *Visie op de wetenschap.* De overheid moet een langetermijnvisie op de wetenschap formuleren, in het besef dat fundamenteel onderzoek aan de basis van innovatie staat. De academische gemeenschap moet zich daarvoor actief inzetten.

9. *Implementatieplan.* De overheid moet een commissie instellen met als opdracht het opstellen van een implementatieplan voor de Nederlandse wiskunde voor de middellange termijn. Het plan formuleert concrete actie voor de wiskundigen in hoger onderwijs en onderzoek, in basis- en voortgezet onderwijs, en in maatschappij en innovatie. Het stelt meetbare criteria vast, die de overheid in staat stelt het effect van extra financiering te beoordelen en deze financiering na bewezen succes structureel te maken.

# 1

# The Mathematical Sciences



## 1.1

# Mathematics: Identity and Perception

It has been 400 years since Galileo formulated his maxim that “the book of nature is written in the language of mathematics”. Written half a century before Newton formulated his laws, these visionary words have proven true to an astounding degree. Besides having deep connections with physics, the mathematical sciences are nowadays the indispensable foundation supporting science, technology, the social sciences, humanities, and much of society itself. The methods of physics, chemistry, computer science, economics and logistics are founded on the mathematical sciences. Progress in mathematics translates into increased efficiency and new possibilities in other fields.

*Mathematics uncovers patterns through logical reasoning.*

Mathematics uncovers patterns through logical reasoning. It deduces far-reaching consequences and reveals vast generalizations. Its universal applicability and power explains the role of mathematics as the common language of scientific thought. Much of the educated public remains unaware that mathematics is always innovating. It does not consist of a fixed set of tools that are ready for application. Every day new theorems are proved, new concepts are developed, and new applications emerge. The adaptation of existing knowledge to new contexts and the discovery of new knowledge go hand in hand, in a never-ending cycle.

The mathematical sciences tend to be invisible, but they are present wherever computers are used, data are available, physics or chemistry is applied, geometry is significant, statistics is employed, processes or designs are optimized. No science has a broader reach than the mathematical sciences. Especially the advent of computational science and the recent data explosion have led to an enormous increase in the relevance and impact of the mathematical sciences.

## 1.2

# The Mathematical Sciences in 2025

*No science has a broader reach than the mathematical sciences.*

In 2013, the National Research Council of the United States published *The Mathematical Sciences in 2025* [NRC 2013], a comprehensive analysis of the field of mathematics, broadly construed. The authors consistently use the term “mathematical sciences” to emphasize the increasing reach of the field.

The document paints a picture of a vibrant and evolving field, moving away from the disciplinary thinking of the 20th century, with its emphasis on “pure” and “applied” mathematics. In the 21st century much of science will be built on a broader and more versatile mathematics, practiced by mathematical scientists who operate in different modes across a spectrum between proving theorems, solving problems, developing algorithms, and designing and analyzing models, as need arises: “Mathematical sciences work is becoming an increasingly integral and essential component of a growing array of areas of investigation in biology, medicine, social sciences, business, advanced design, climate,finance, advanced materials, and much more. This work involves the integration of mathematics, statistics, and computation in the broadest sense, and the interplay of these areas with areas of potential application; the mathematical sciences are best conceived of as including all these components. These activities are crucial to economic growth, national competitiveness, and national security. This finding has ramifications for both the nature and scale of funding of the mathematical sciences and for education in the mathematical sciences” [NRC 2013, pp. 110-111].

A major catalyst for this transition is the increasing mathematization of science and technology through the proliferation of computational science and data-based research. This, together with advances in the study of complex systems, networks and uncertainty, contributes to a broader reach and greater impact of the mathematical sciences. Accompanying this development is a blurring of the boundaries between the mathematical sciences and other fields, as multidisciplinary science searches for a common ground in an abstract methodology. At the same time, it is crucial for the health of the mathematical sciences to maintain and develop a strong core, which secures the internal coherence of the field. It is the nature of mathematics to work towards abstraction, to establish proof, and to develop a coherent core. This core is also the key to its strength, providing the foundation on which much of 21st century science and engineering will be built.

*A demand is arising for a new class of mathematically trained workers.*

As more and more fields become mathematics dependent, a demand is arising for a new class of mathematically trained workers. This will require the training of a large mathematically literate workforce. It motivates a rethinking of mathematics education, including the teaching of mathematics to students in other disciplines, and should lead to the introduction of new degrees. Graduates in mathematics should possess a broader overview knowledge, be able to communicate well with non-mathematicians, and have experience with mathematical software and computation. With abstraction as their main tool, they are driven by curiosity, inspired by practice, and accountable to society.

## 1.3

## The Mathematical Sciences in the Netherlands

Mathematical research in the Netherlands, despite its modest size, is of a very high quality. It has a long-standing tradition of excellence in several areas [NWO 2011a], which play a prominent role in current international trends.

The mathematics student enrollment gradually declined starting in 1989 but has been growing again since 2003, reaching unprecedented high levels in 2012 and 2013. Since the early 1990s, mathematics staff numbers at Dutch universities have declined by about forty percent.

In the same period, Dutch mathematics took the initiative to organize its master level teaching and research and give an impetus to focus areas. National mathematics clusters were established, with membership transcending universities, to showcase subdisciplines in which our country enjoys strong concentrations. Research within the clusters covers the whole range of activities from fundamental work to modeling and computation. In 2011 the cluster program was evaluated by an international panel [NWO 2011a], which concluded that the quality of research, organization and development attains an excellent level, and recommended to continue the clusters and consolidate funding, to stimulate internal competition through allocation of funds, and to increase involvement of non-local nodes in new hires. The clusters were continued for two years in 2011, and renewed again in 2013.

At the administrative level, the Dutch mathematics community organized itself in the “Platform Wiskunde Nederland” (PWN). In PWN, the Royal Dutch Mathematical Society and the Dutch Society of Mathematics Teachers are joining forces in matters of education, research and innovation, and in speaking with one voice towards government, funding organizations, industry and the public at large.

Many other initiatives are notable, such as intensifying contacts with high school teachers, the introduction of *Wiskunde D* modules in secondary education, the popular website

*Wiskundemeisjes* (*Math girls*) aimed at high school students, and many local outreach activities.

The success of the Dutch mathematics community over the past decades is all the more remarkable in view of its small size. Its consolidation and expansion over the coming decade is a challenging task in view of the globalization, the strong international competition, and the international trend of mathematics research crossing subdisciplinary and disciplinary frontiers. This process requires sustained long-term investment and political commitment.

### 1.4

## The Value of Mathematics

At the request of PWN, Deloitte carried out an investigation with the aim of quantifying the impact of the mathematical sciences on the Dutch economy [Deloitte 2014]. Similar studies were carried out in the UK for physics and for the mathematical sciences [Deloitte 2012, 2013].

Deloitte estimated the mathematics intensity of jobs in our country and clustered them according to 39 industry categories, using data of Statistics Netherlands. They then applied input-output analysis to determine the overall contribution of the mathematical sciences to the Gross Value Added of the Netherlands. The analysis distinguishes between direct, indirect and induced effects. Deloitte also provides a qualitative assessment of the relation between mathematics education and mathematical skills on the one hand and national economic competitiveness on the other hand. Their main conclusions are the following.

The mathematical sciences are used in the Netherlands by the full-time equivalent of about 900,000 highly educated professionals. These range from scientists who use mathematics all the time, to bankers and physicians, who sometimes use mathematics to compute the value of assets or to interpret medical tests.

These 900,000 jobs not only generate direct income for the employees involved, but also for people employed in industries that supply goods or services to organizations where mathematical sciences practitioners work, and in the businesses where these practitioners spend their own income. These so-called indirect and induced effects are estimated to create another 1.4 million jobs, resulting in a contribution of the mathematical sciences toward total employment of about twenty-five percent. Since these are high-income jobs, the economic contribution of the mathematical sciences represents about thirty percent of Dutch national income.

*A strong foundation for the mathematical sciences is a critical element in the success of an advanced economy.*

Deloitte notes that a strong foundation for the mathematical sciences is a critical element in the success of an advanced economy. Better mathematical skills correlate with a more competitive economy and a higher standard of living. Moreover, the importance of the mathematical sciences to society is likely to increase substantially in the coming decades with the revolution in computational science, big data, and business analytics.

Our country runs the risk of missing out on these trends and losing its competitiveness as a nation, because the number of science and engineering graduates has dropped to the lowest level in Europe. At current graduation rates, 400,000 direct, indirect and induced jobs could disappear. A key first step for the Netherlands will therefore be to expand and improve mathematics education across many fields of study.

### 1.5

## Vision Document

The present document is different in scope and purpose from the *Masterplan toekomst wiskunde* (*Masterplan for the future of mathematics*) [NWO 2008] and its update, the *Masterplan toekomst wiskunde 2.0* [NWO 2011b]. It takes major trends in society and science as its starting point; it identifies grand challenges for mathematicians worldwide, indicating important areas where Dutch researchers may play a prominent role, for the benefit of the mathematical sciences, other sciences and society; and it outlines the road ahead for education, research and innovation in the medium term, say, to 2025. The document is about challenges, opportunities, strengths, weaknesses, self-awareness, and public image.

*The document is about challenges, opportunities, strengths, weaknesses, self-awareness, and public image.*

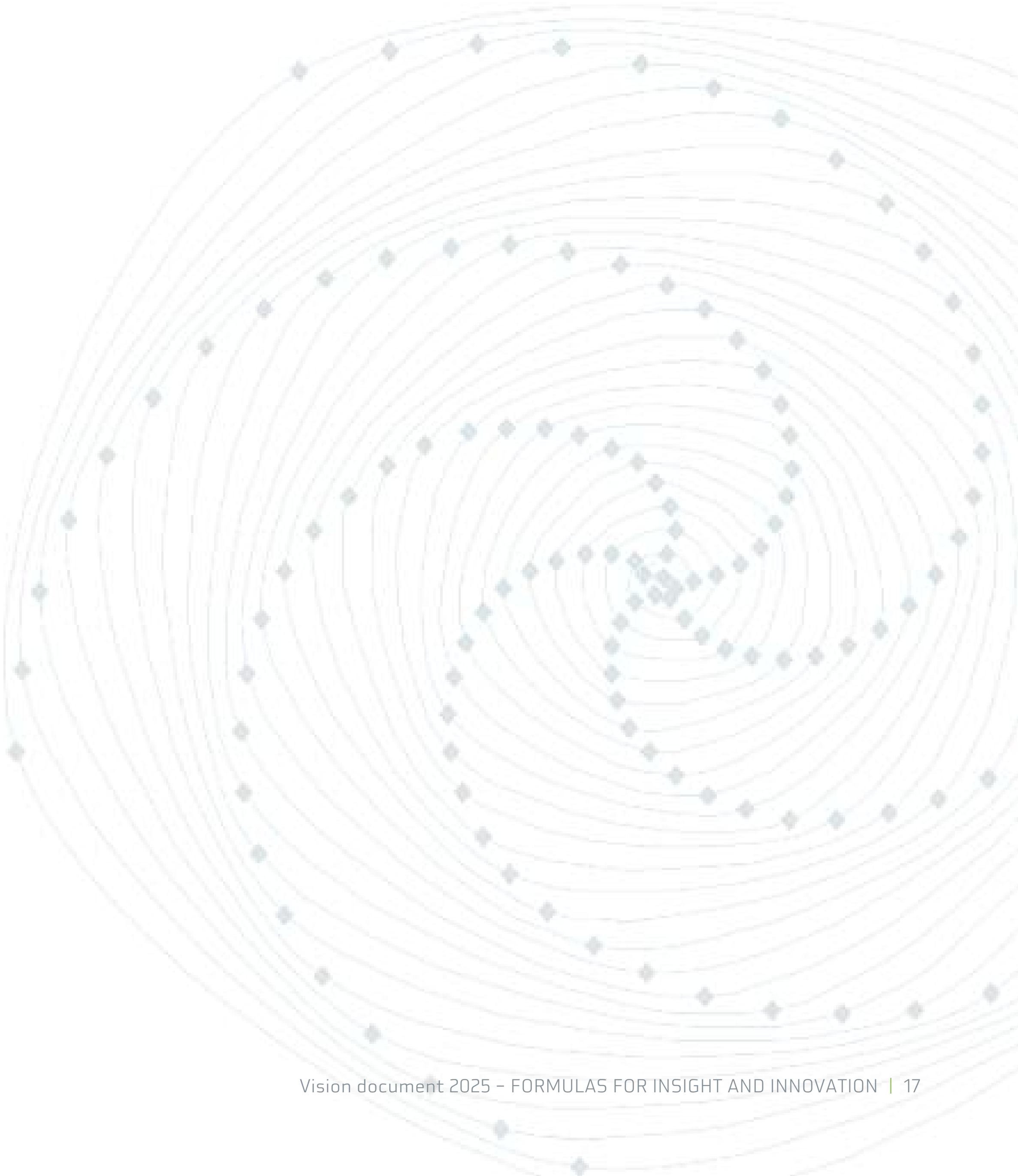
This document, the study by Deloitte, and a book with success stories of Dutch mathematics were instigated by PWN. The three documents send a strong signal towards the Ministry of Education, Culture and Science, the Ministry of Economic Affairs, the Netherlands Organisation for Scientific Research (NWO), deans of science faculties, directors of mathematics departments, and other stakeholders, including the mathematical scientists themselves. By the summer of 2014 the government will publish a white paper with a vision of science and of the research infrastructure in the Netherlands. The publication of the current document is well timed to inform this process.

In the following pages, we discuss six trends in society and science that influence the mathematical sciences, and present five grand mathematical challenges. We then analyze the mathematical sciences in the Netherlands in terms of their strengths, weaknesses, opportunities and threats on three dimensions: higher education and research, primary and secondary education, and society and innovation. We conclude this vision document with a number of recommendations.

## 2

# Trends in Society and Science

We will discuss six major societal and scientific trends that influence the mathematical sciences. These trends shape how we do mathematics, what questions we investigate, and how we teach our students.



2.1.

## Globalization

The world is becoming very small. Nowadays, people easily move between continents to study or work, and the internet facilitates long-distance education and collaboration. This has a profound impact on how we educate young people and how we do research.

*Globalization has far-reaching effects in science and education.*

Globalization has far-reaching effects in science and education. It is estimated that 900,000 students from China and India will study abroad in 2025. In many fields of graduate study at American universities, Chinese students form the majority. Georgia Tech founded a logistics institute in Singapore over ten years ago, New York University has opened branches in Shanghai and Abu Dhabi, and also the Sorbonne has opened a branch in Abu Dhabi. After international competitions, Cornell and the Technion are founding a new innovation campus in New York City, and MIT, Delft and Wageningen will establish a technology center in Amsterdam. These developments will only strengthen, as universities nowadays compete for young talent worldwide. The Netherlands is in a good position to profit from this trend.

Globalization is to a large extent made possible by network technology. We live in a networked society. Networks shape our lives in ways that were unimaginable a single generation ago. Millions of people in the Netherlands are connected through transportation networks, energy grids and the internet. Billions of people exchange information through social networks. We book our flights through the internet, plan our travels using the world-wide web, and find our next job using LinkedIn. Important developments find their way quickly to file sharing systems like YouTube, and people share their lives through social media like Facebook and Twitter. At the same time, personal information is readily available to many parties. We leave a digital footprint that may haunt us for the rest of our lives. Privacy and security are major issues, which pose formidable mathematical challenges.

The networked society is also entering the classroom, from primary schools to universities. This offers many opportunities. In primary and secondary education, teachers often have to occupy themselves with the slow learners, leaving more gifted pupils to their own devices. The latter are now able to access advanced learning tools available on the internet; see, for example, www.dedigitaletopschool.nl. In academia, massive open online courses can be used to replace courses or to supplement the regular teaching program. With extra office hours and one-on-one meetings, such courses could profoundly change the way university curricula are organized, particularly at the master and PhD level, where student numbers are small and courses specialized.

### Six degrees of separation

**It's a small world after all. According to a popular theory, anyone on earth is six or fewer handshakes away from anyone else. In mathematical terms, this means that in a graph with nodes corresponding to human beings and edges to handshakes, there is a path of at most six edges between any two nodes. This is nearly impossible to prove in practice, but it can be reliably determined for virtual social networks. The average degree of separation for users of Twitter and Facebook turns out to be 3.43 and 4.74, respectively. These low numbers largely account for the rapid spread of news, ideas and memes on social networks.**

## 2.2.

# Computation

The proliferation of computing in the past decades has transformed both the mathematical sciences and its application areas.

Starting with the latter, computational science – as opposed to theoretical and experimental science – plays an ever more prominent role by providing the virtual laboratory in which science is practiced. The computational paradigm synthesizes knowledge specific to the area, programming constructs from computer science, and methods and theory from the mathematical sciences.

*Computational science plays an ever more prominent role by providing the virtual laboratory in which science is practiced.*

Computational science is used in designing and testing engineering parts in their environment. The behavior of the phenomena is complex but predictable, allowing high-precision calculations. Examples are virtual wind tunnels used in the design of aircraft, and circuit simulation environments used by chip manufacturers.

In other cases, simulations are used for prediction and exploration. Models derived from accepted physical laws may still exhibit complex and unpredictable behavior. The computational environment allows one to observe and describe the types of behavior that occur. Examples are configuration studies in biomolecules, epidemiological models for the spread of diseases, and climate simulation.

Computational science methodologies have been extended to conceptual models beyond the traditional physical setting. Computational models are developed for social and societal systems and other fields that have not been traditional subjects of mathematical analysis. Examples involve public transport users, goods in transport networks, economic players, but also flocking in birds and fish and collective cell behavior during embryonic development. These models are often phenomenological systems consisting of decision-making agents or cellular automata, and the object of study is the large-scale coherent dynamics of a large number of interconnected individuals.

The ubiquity of computation is transforming the mathematical sciences themselves as well, by providing new questions and challenges. All of computational science and many more applications that involve computation depend on, and are driving fundamental research in, for example, scientific computing, optimization, stochastics, and number theory. This has given us faster algorithms that augment our capabilities to do accurate large-scale computations. In fact, algorithmic progress has often been responsible for larger speedups than improvements in hardware.

### Software improvement

**Moore's Law states that the number of transistors on a chip doubles every two years. A less-recognized fact is that the speed improvement in computers due to better hardware is dwarfed by algorithmic improvements in software. According to optimization expert Martin Grötschel, a benchmark production planning problem modeled in terms of linear programming would have taken 82 years to solve in 1988, while it could be solved in a minute in 2003, a factor 43 million faster. Hardware improvements account for a factor of 1,000, software improvements account for the remaining factor 43,000.**

The rise of computation is responsible for a broader cultural shift in mathematics. The focus has gradually moved from the classical existence question (does a solution exist?) towards the computability question (can we efficiently find one?). This *explicit* – as opposed to *existential* – approach was long considered the domain of applications, but is becoming significant in the theoretical domain as well.

A related trend is the increasing use of computer experimentation within mathematics, which changes the way research is done. Innovation in theory frequently comes from inspiration gleaned from painstakingly computed examples. Mathematicians fruitfully make use of the possibilities of algorithmic computation, moving from abstract thought to experiment and back.

*Science is moving into a new age of numbers.*

### 2.3.

## Big Data

Next to the ubiquity of computation, the unprecedented growth in the availability of data is a second major driver of the increased reach of the mathematical sciences.

Science is moving into a new age of numbers. *Big data* refers to data-intensive applications in which the data sets themselves present a challenge to efficient computation due to an excessive data volume, data flow rate or data heterogeneity. The availability of data in all aspects of society, business and science is growing rapidly due to the penetration of data-recording devices from smart phones and smart power meters to microarrays, fMRI machines and satellites. Compounding this are increased connectivity due to internet, smart grids and the "internet of things" with associated logging of online transactions and browsing behavior; higher resolution of measurement devices; and growing use of simulation. Data-intensive applications include gene research, forensics and security, remote sensing, web analytics, finance, drug development, transportation, and marketing, among many others. At the same time, there is a trend in society and technology toward increasing reliance on expert-informed planning, risk analysis, and analytical tools for decision-making, and hence on mathematical methods.

Overall, the accessibility of data is driving a trend in data-intensive applications towards problems that fall outside the boundaries of classical statistical inference. In the words of Bradley Efron, renowned Stanford-based statistician, we have entered a new era in the history of statistics, the era of scientific mass production. The questions accompanying modern data analysis problems have evolved beyond the conceptions of statisticians at the beginning of the twentieth century. In addition to the size of data sets, the

### Customer data

**Big data mean big profits. With a web shop as large as Amazon.com, customer data are a potential goldmine. By registering the clickstream data and purchase history of its 130 million customers, the shop has accurate data about what online customers want to buy and how they search for it. Amazon.com sells these data to marketeers, who use it to devise smart algorithms that make split-second decisions about where to buy an advertisement and for what price.**

complexity of the information we want to extract from data is rapidly growing as well. In contrast to more classical statistical settings, the trend nowadays is often to learn high-dimensional, complex structures from the data. Accompanying this change is the continuing evolution of massively parallel computers, with their potential to facilitate large-scale data analysis.

In response to this trend, several Dutch universities are presently starting data science institutes.

### 2.4.

## Math Inside

*The mathematical sciences are pervading science and society.*

Mathematics has often been ahead of its applications. Many results obtained for their own sake by mathematicians whose driving forces were abstraction, elegance and beauty, later turned instrumental in solving real-world problems. Examples include the application of elliptic curves in cryptography, of finite groups in constructing error-correcting codes, of geometry in theoretical physics, of harmonic analysis in imaging, and of stochastic analysis in the modeling of financial markets.

In the opposite direction, the real world has been a continuous source of inspiration for mathematics. For instance, the planning of the Soviet economy in the 1930s and the logistics of warfare in the 1940s gave rise to linear programming. This became the most powerful tool of operations research, and eventually developed into a theory of its own with many applications in other areas of mathematics.

In the past decades, the interplay between mathematics and its fields of application has intensified, with the invention-innovation cycle getting shorter over time. The borderlines with areas such as computational science, mathematical physics, biomathematics, operations research, economics and financial mathematics are blurring. The use of mathematical concepts and techniques is extending into the social sciences and the humanities.
The mathematical sciences are pervading science and society, providing the foundation for computational simulation and data analysis. Scientists from other disciplines value the collaboration with mathematicians, who think in structures, who find the essence of problems by abstracting away from incidental aspects, and who are often able to contribute deep and unexpected insights. In reverse, the new use of mathematics poses new problems and inspires new research.

### Elliptic curve cryptography

**An elliptic curve is an algebraic curve of the form $y^2 = x^3 + ax + b$. Elliptic curves had been at the core of mathematics for over a century, when – a little over 25 years ago – they turned out to be well suited for cryptographic purposes. It is easy to compute a multiple nP of an elliptic curve element P for a given number n, but there is no efficient method of finding n when given P and Q = nP. In cryptographic applications an eavesdropper would need to solve such an equation in order to decode an encrypted message. What was an abstract concept is now used to safeguard our online communication.**

This is a recent and major development, which has given a new stature to the field. With the advance of globalization and the development of ever more sophisticated technologies, the need for and impact of mathematical research is becoming more and more pronounced. A flourishing mathematical community is a vital commodity for a dynamic society and an innovative economy. In order to be able to continue renewing itself from within, it needs critical mass and academic freedom. The relative unpredictability of applications reinforces the importance of fundamental research and of maintaining a healthy core. The mathematics core is a critical part of the knowledge infrastructure.

## 2.5.

## Coherence

*The classical division of the mathematical sciences in subdisciplines is fading.*

The classical division of the mathematical sciences in subdisciplines is fading. Some of the most exciting advances reveal connections between previously unrelated subdisciplines. Such cross-fertilization has always existed, but its importance is growing steadily.

Exemplary to this trend are the seven Millennium Prize Problems of the Clay Mathematics Institute, presented in 2000 to mark the grand challenges for the 21st century. The seven problems have a strongly interdisciplinary character or present deep problems that cross over subdisciplinary borders. The only one that has been solved to date is the Poincaré conjecture. The solution of this purely topological problem surprisingly used partial differential equations and differential geometry.

The trend is just as strong in more applied areas. For instance, the investigation of complex dynamical systems relies on the combination of non-linear analysis, stochastic modeling, and scientific computing. The theory of large deviations, a cornerstone in probability theory for which Srinivasa Varadhan was awarded the Abel Prize, helps to explain the occurrence of rare events in statistical physics, finance, and traffic congestion.

The growing coherence of the field is reflected in an increase in research collaborations, causing a gradual shift from single-author to joint publications. To be successful at the frontiers of research, mathematicians increasingly need to broaden their perspective, have a good sense of global developments, and be able to communicate and collaborate with researchers from diverse areas.

### Millennium Prize Problems

**In 2000 the Clay Mathematics Institute announced a prize of one million dollars for the solution of each of seven Millennium Prize Problems. The focus is on classical questions that have remained unsolved for many years. One of these problems, the Riemann hypothesis, already appeared on a similar list presented by David Hilbert in 1900. Until now only the Millennium Prize Problem on the Poincaré conjecture has been solved, by the Russian mathematician Grigoriy Perelman. In 2006 *Science* recognized his achievement as the scientific breakthrough of the year. Perelman declined the prize, however.**

## 2.6.

# Accountability

Mathematicians have become aware of the need to be accountable for their work to society at large. They are faced with an increased demand for justification of their work outside its own context. Nowadays, grant programs often require a utilization paragraph, and a substantial portion of research funding is being allocated to specific societal problems. This encourages mathematicians to reflect on the position and implications of their work in a broader context and to become responsive to societal challenges.

Utilization is different from direct applicability. Fundamental work that is not driven by immediate demand is often abstract in nature but still inspired by questions rooted in reality or in other sciences. It strengthens the core of the field, expands human knowledge, and carries the potential for applicability in the long run. It contributes to innovation, albeit not necessarily to today's gross national product.

*A flourishing mathematical community is a vital commodity for a dynamic society and an innovative economy.*

Accountability has many forms. It also refers to the ability to communicate the fascination for discovery and innovation. Mathematicians increasingly recognize the importance of outreach activities. There is a sustained effort to inform general audiences about recent discoveries in mathematics and their usefulness in society. The BBC series on the history of mathematics is shown in many high school classrooms. In the Netherlands, leading mathematicians are lecturing at Lowlands and regularly explain science issues on television. The TV program "Eureka" aims at the popularization of mathematics. Mathematical olympiads attract great numbers of talented students, some of whom win gold medals at the international level. These efforts show that the mathematical sciences are a thriving field of science, strive to improve the layman's image and appreciation of the field, and may well explain the recent growth in mathematics student enrollment.

### Lowlands University

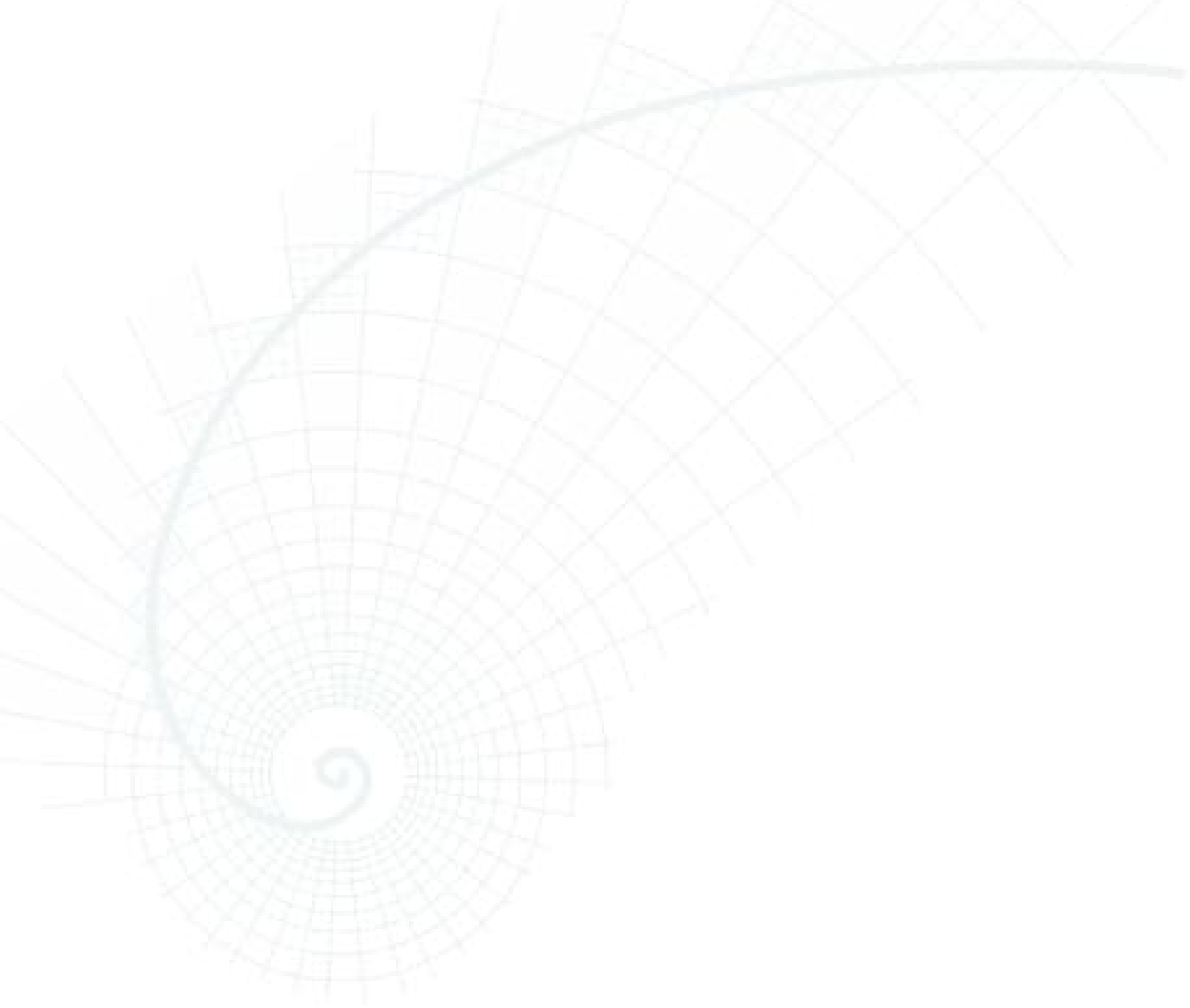
**An unusual science outreach activity is Lowlands University. Prominent scientists are invited to address a large audience at Lowlands, a popular summer music festival. They receive star treatment and speak to hundreds of young festival goers about their work. Prominent Dutch mathematicians had the honor of joining Lowlands University, including Hendrik Lenstra on the mathematics behind one of Escher's prints (2007) and Lex Schrijver on shortest paths, traveling salesmen and railway schedules (2012).**

# 3

# Grand Challenges in Mathematics

The mathematical sciences are in rapid development. Breakthroughs are taking place at the interface of classical subdisciplines, with impact reaching beyond the mathematical sciences. Current trends in science and society provide new interdisciplinary challenges with a prominent role for the mathematical sciences.

We formulate five grand challenges recognized by mathematicians worldwide, which are broadly in line with the 2011 Dutch Research Agenda of the Royal Netherlands Academy of Arts and Sciences (KNAW) [KNAW 2011].



## 3.1.

# The Langlands Program

The Langlands program is an ambitious mathematical program that ties together major but seemingly distant areas of algebra, geometry and analysis. It was put forward by Robert Langlands in the 1970s and is based on the principle that symmetry imposes order. This principle is powerful in many contexts and plays a key role in the behavior of physical systems. The program predicts that also the behavior of prime numbers is, to a large extent, governed by symmetry, which would have revolutionary implications in number theory.

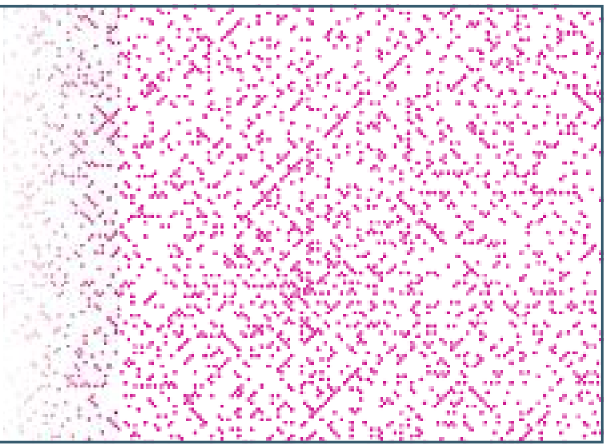
Since Langlands formulated his program, substantial progress has been made in its development. The most celebrated example is the proof of Fermat's last theorem by Andrew Wiles. Recently a geometric version of the program has been proposed, where the role of numbers is replaced by functions on a curve. This program has startling prospects, most prominently in providing a bridge between geometry and quantum physics.

The Langlands program involves a rich mixture of ideas, including analogies between seemingly disjoint areas of mathematics and physics. It provides the ideal context for innovative advances in all areas involved. The full understanding of the Langlands program and its generalizations poses a grand challenge in mathematics for many years to come.

*Breakthroughs are taking place at the interface of classical subdisciplines.*

### Prime number symmetry

**A natural number larger than 1 is prime if it has no other divisors than 1 and itself. The structure of the set of prime numbers is a core subject of number theory. Each natural number can be expressed uniquely as a product of prime numbers, but we have no fast algorithm for finding those prime factors. Many questions about prime numbers remain unanswered. In particular, there appears to be no regularity in how the sequence of primes is situated amongst the other numbers. Yet, the Langlands program surprisingly uses geometry to relate the sequence of prime numbers to symmetries of curved space.**



## 3.2.

# Complex Systems

*There is a multidisciplinary effort to seek common ground in a mathematical theory of complex systems.*

Complex systems arise when a large number of comparatively simple systems interact, such that their collective behavior cannot be inferred from the individual components. Examples are bees in a hive, cells in an organism, and investors in a market. Their investigation is a relatively recent multidisciplinary effort, which presents many mathematical challenges.

The dynamics of complex systems are not well understood. How do global patterns emerge from the behavior of individuals? Why are they resilient to some external forces and yet vulnerable to others? To what degree are they predictable? These all pose substantial mathematical problems.

There is also the challenge of scale. The coupling of systems at different space and time scales is a major issue in the modeling and analysis of complex systems. A biological organism for instance is governed by interactions at molecular, cell, tissue and organism levels. Challenges include the representation of cross-scale interactions and feedback mechanisms, as well as computational methods for treating scale.

Critical societal infrastructures like transportation, communication and energy networks can also be viewed as complex systems. The decentralized decisions in such networks often make their behavior unpredictable. The complex systems perspective may lead to valuable insights in their resilience when operating at maximum capacity or even beyond.

There is a multidisciplinary effort to seek common ground in a mathematical theory of complex systems. The subject holds great potential, in developing new mathematics for studying and exploiting such systems and in the process to extend their impact to society and policy making.

### The economy as a complex system

**The financial crisis of 2008 has greatly stimulated the interest of economists and mathematicians in new models and techniques to describe and predict the economy. The traditional assumption that the economy is a near-equilibrium, almost linear system, with any fluctuations being the effect of shocks that dampen out over time, appears to be invalid. Economic processes form rather a highly nonlinear complex system, characterized by boom-bust patterns, positive feedbacks, and herd behavior. Using newly available resources of large-scale computing and big data, mathematical economists are beginning to devise new approaches, based on complex systems notions, predicted from and tested on empirical and experimental data sets.**

## 3.3.

# Stochastic Modeling

The world is full of randomness. Elementary particles move unpredictably, stock markets rise and fall, and streams are disturbed by turbulence. To compensate for our incomplete understanding of such systems, stochastic modeling is used. The development of accurate stochastic models and their analysis is one of the grand challenges of mathematics, with applications in for instance physics, biology, finance, and mathematics itself.

In physics, stochastic models are used to model and analyze the behavior of large systems that are microscopically random, like fluids and gases. In finance, they are used to estimate the risk of financial products. The recent worldwide financial crisis has shown the need for better risk management strategies. Their development requires profound mathematical expertise, a fact that is increasingly recognized by banks and insurance companies. Also, large networks for communication, computers, energy and manufacturing are complex stochastic networks. The challenge is to design networks that are able to make reliable decentralized decisions without human interference. This generates exciting questions in for instance rare event simulation, queuing, optimization, control, and game theory.

*The world is full of randomness.*

In mathematics itself, stochastic techniques are increasingly used. They have led to progress in solving partial differential equations. In number theory, connections between the distribution of prime numbers and the properties of random matrices may eventually lead to the confirmation of the Riemann hypothesis. In the mathematics of algorithms, randomized algorithms yield efficient solutions at the price of practical instead of absolute certainty.

### Emergency planning

**The timely arrival of an ambulance can be a matter of life and death. Uncertain factors such as traffic and location of the accident are hardly taken into account in existing planning methods. Dutch mathematicians are working with ambulance service providers on new planning methods that use stochastic modeling of uncertain factors. By anticipating the need for ambulances and a dynamic relocation of available vehicles, the method ensures optimal coverage of the region at any time.**

### 3.4.

## Efficient Computation

The grand challenge of the mathematics of algorithms is to determine the limits of computability, as a function of the time bounds imposed and the model of computing chosen.

For some problems, clever algorithms give fast solutions. Dijkstra’s algorithm quickly finds the shortest route between two cities amongst the huge number of possibilities. “Fast” here means that the time required is polynomially bounded in the problem size. No such fast algorithms are known for many other problems, such as finding a feasible railway timetable, or factoring an integer into primes – and much of cryptography depends on the assumption that no fast method for doing so exists. Trying all possibilities one by one may run into millions of years.

*A further-reaching attempt is to allow algorithms to make use of quantum mechanical effects.*

The P versus NP problem, one of the Millennium Prize Problems, asks if this distinction between easy and not-so-easy problems is fundamental. If one can quickly check the feasibility of a timetable, can one also quickly find a feasible timetable, if one exists? The answer has far-reaching consequences. An affirmative answer would imply that a host of highly relevant computational problems are easy. A negative answer would confirm that all of these problems are inherently hard.

Although the question remains open, it is generally believed that P and NP are not equal. Still, we need to solve hard problems in practice, and the question is how to deal with them. The theory of algorithms provides many tools for this. Some hard problems can be approximated fast; others have algorithms that are fast enough for practical large-scale instances; for still others there exist fast heuristics that work well most of the time.

A further-reaching attempt is to broaden the model of computation by allowing algorithms to make use of quantum mechanical effects. Although quantum computers are yet to be built, some hard problems have fast quantum algorithms, the prominent example being Shor’s algorithm for factoring integers into primes. It is conjectured, however, that quantum computing will not turn all hard problems into easy ones, but likely will help with just a few of the problems that we care about.

### Shortest paths

**Dijkstra’s algorithm for finding a shortest path between two nodes in a network is a classic example of efficient computation. It was conceived by Dutch mathematician and computer scientist Edsger Dijkstra in 1956. It is based on the idea that any subpath of a shortest path must also be a shortest path. The elegant and powerful algorithm is one of the most widespread export products of the Netherlands. It finds its way in state-of-the art navigation software, routing protocols for computer and communication networks, and traffic control software.**

## 3.5.

# Data Science

With the rise of big data, the grand challenge is what the mathematical sciences can contribute to generating maximum information from available data. Big data research relies heavily on statistics and numerical methods. Statistical methods are used for analyzing data and discovering correlations; numerical algorithms are used for efficient data manipulation and model reduction. A powerful new technology is topological data analysis, which uses the shape of the data set to identify its essential structure. It leads to visualization methods that allow for a quick analysis of data and for the identification of interesting phenomena that might otherwise remain hidden.

Statisticians are challenged to develop principles and algorithms for learning complex objects and inferring knowledge from large data sets. The methodology of the early twentieth century has to be revisited and adapted to be useful in modern settings, or replaced by completely new approaches. In particular, scalable algorithms must be devised for efficiently handling large quantities of data in massively parallel computing environments.

*The methodology of the early twentieth century has to be revisited and adapted, or replaced by completely new approaches.*

A new theory needs to be built for developing and assessing statistical methods and learning algorithms tailored to big data questions. Classical statistics has well-developed methods for analyzing the effectiveness and efficiency of procedures and for quantifying the uncertainty in statistical inferences. For large-scale problems such methods are largely unavailable. Developing a mathematical theory for data science is a key challenge for the years ahead.

### Making new connections

**Better mathematical tools for analyzing complex data sets can lay bare relationships that are too subtle to recognize with traditional techniques. *Nature* (February 2013) reports on the use of techniques from topology to uncover patterns in old data sets that were invisible before. The results include**

- **discovery of new subsets of breast cancer patients that are more responsive to targeted therapy, based on genetic data,**
- **identification of political subgroups within the Democratic and Republican parties, based on 22 years of voting data in the US House of Representatives, and**
- **subdivision in thirteen different roles instead of the traditional five for professional basketball players, based on performance statistics of the 2010-2011 NBA season.**

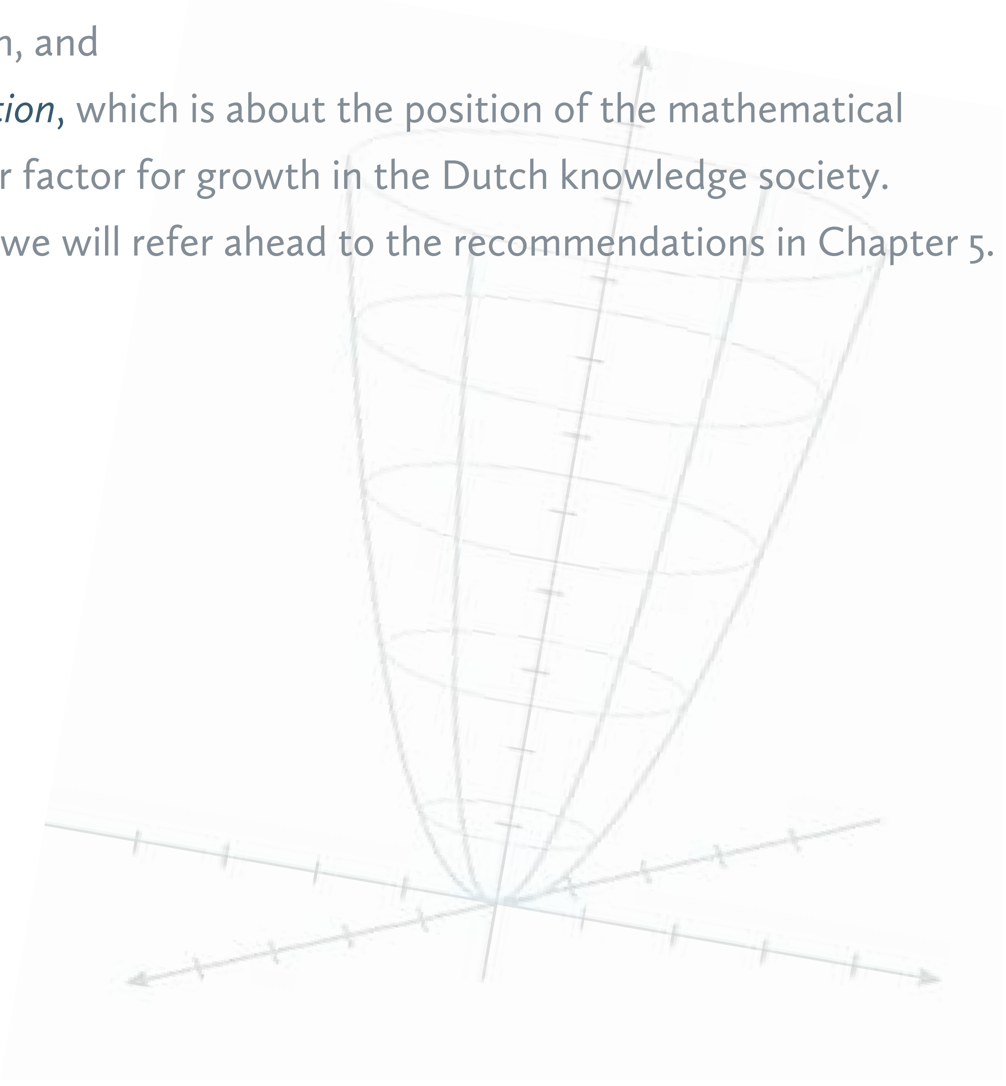
# 4

# SWOT Analysis

We will now discuss the extent to which the Dutch mathematical community is well positioned to respond to the trends discussed above and to take up the challenges for the coming decades. Its strengths, weaknesses, opportunities and threats will be analyzed along three dimensions:

- *higher education and research*, which form an inseparable unity for any mathematician in academia,
- *primary and secondary education*, which are facing a number of pressing issues of their own, and
- *society and innovation*, which is about the position of the mathematical sciences as a major factor for growth in the Dutch knowledge society.

Where appropriate, we will refer ahead to the recommendations in Chapter 5.



**STRENGTHS**

- open community
- focus, coherence and self-organization
- increasing student numbers
- high quality of education, research and applications
- external orientation, universal problem-solving capability
- outreach activities

**WEAKNESSES**

- inaccessibility of subject, invisibility of results
- low participation of women and minorities
- lack of breadth in PhD education
- erosion of teacher education
- no tracks fostering excellence in primary schools

**OPPORTUNITIES**

- broadening reach and impact
- globalization and information revolution
- role of university in teacher education
- public at large educated and fascinated
- improved attitude of students
- funding by industry and other sources

**THREATS**

- small volume, high and increasing workload
- funding of fundamental research under pressure
- funding of research top-down and demand-driven
- math competence in primary and secondary education falling back

### 4.1.

## Higher Education and Research

*open community*

The Dutch society is open and well organized, and maintains a high standard of education and research. Dutch academia is a welcoming environment for foreign talent, and Dutch mathematics is no exception. Many mathematics institutes have a high ratio of foreign to domestic staff and are successful at attracting talented PhD students and postdocs from abroad. International exchange is important for cross-pollination of scientific knowledge and for fostering Dutch connections within the global scientific network.

*focus, coherence and self-organization*

The bundling of various academic mathematics teaching programs creates a dynamic environment for both students and teachers. Examples are the joint bachelor programs of Delft and Leiden and of the two Amsterdam universities. The universities of technology in Delft, Eindhoven and Twente have a joint master program, which is part of their 3TU Applied Mathematics Institute.
A successful initiative is Mastermath, which offers master courses on a national level. It includes all Dutch universities as well as the Dutch Network on the Mathematics of Operations Research and the Dutch Institute of Systems and Control. Mastermath has extended its program with high-level courses that are also of interest to PhD students.
Since 2005 the Netherlands Organisation for Scientific Research (NWO), with the help of the Ministries of Education, Culture and Science and of Economic Affairs, has initiated national mathematics clusters to provide focus, to ensure critical mass in four broad areas of mathematics, and to stimulate interaction; see the overview below. The clusters are unique to the Dutch scientific landscape. They play a crucial role in enhancing our national pride and identity within the mathematical sciences. In the future, the clusters will need to prove their vitality by showing to be dynamic and adapting themselves to developments of the discipline. *See Recommendation 2.*

The funding of the clusters has been ad hoc. DIAMANT, NDNS+ and GQT received 10M€ over the first five years, the new cluster STAR received 1.5M€ in 2010, the four clusters received 4M€ in 2011 and 4M€ again in 2013. They initially sponsored 24 new academic positions and are funding fifteen new tenure-track positions starting in 2014. *See Recommendation 2.*

A strong sign of the self-organizational capacity of Dutch mathematics is the founding of the "Platform Wiskunde Nederland" in 2010, with financial support from NWO. PWN has active committees on education, research, innovation, publicity, and publications. PWN represents the Dutch community in the mathematical sciences in political issues, e.g. by commissioning the study by Deloitte and the present vision document, and is in the position to strengthen the voice of mathematics in discussions on improving STEM education, i.e., education in the broad field of science, technology, engineering and mathematics.

## Membership and research themes of mathematics clusters

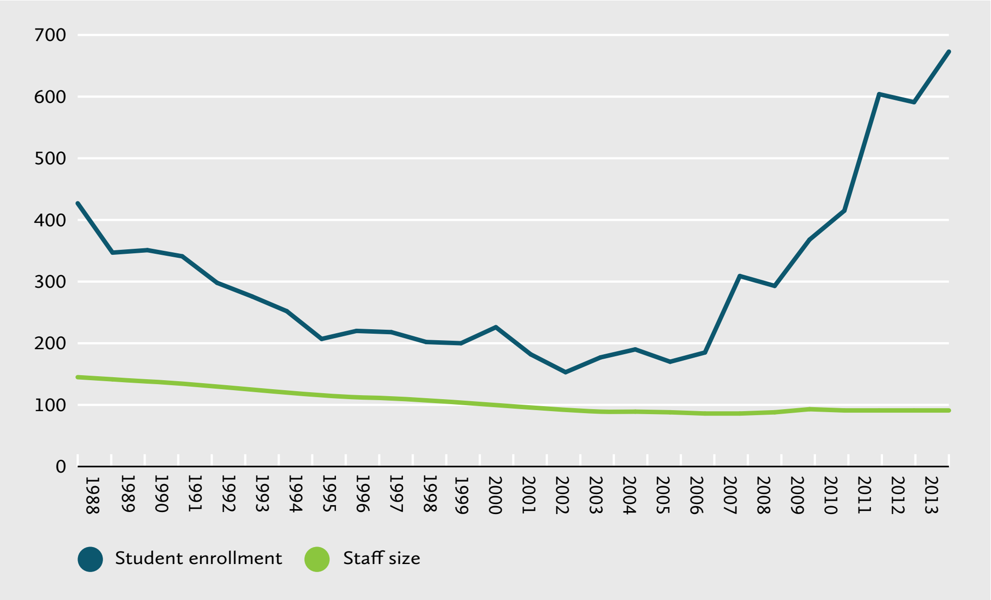
| **Membership** | **DIAMANT** | **NDNS+** | **GQT** | **STAR** |
|---|---|---|---|---|
| Leiden University | ● | ● |  | ● |
| University of Groningen | ● | ● |  | ● |
| Utrecht University | ● | ● | ● |  |
| University of Amsterdam |  | ● | ● | ● |
| VU University Amsterdam | ● | ● |  | ● |
| Delft University of Technology | ● | ● |  | ● |
| Radboud University Nijmegen | ● |  | ● | ● |
| Tilburg University | ● |  |  |  |
| Centrum Wiskunde & Informatica | ● | ● |  | ● |
| Eindhoven University of Technology | ● | ● |  | ● |
| Twente University | ● | ● |  | ● |

| **Research themes** |
|---|
| DIAMANT, Discrete, Interactive and Algorithmic Mathematics, Algebra and Number Theory: discrete mathematics, algebra, geometry, number theory, optimization, algorithms and complexity, coding and cryptology. |
| NDNS+, Nonlinear Dynamics of Natural Systems: bifurcation and chaos, multiple scales, scientific computing, patterns and waves, variational methods, stochastic dynamics, analysis of networks. |
| GQT, *Fellowship of Geometry and Quantum Theory*: algebraic and arithmetic geometry, category theory, geometric analysis, integrable systems. |
| STAR, *Stochastics – Theoretical and Applied Research*: Bayesian non-parametric statistics, non-parametric and semi-parametric statistics, applications of statistics in life sciences and economics; stochastic networks, queuing theory, performance analysis; spatial stochastics, mathematical statistical physics, random graphs; financial mathematics. |

*increasing student numbers*

In 1988, 427 mathematics students entered Dutch academia. Then a gradual decline set in, with an all-time low of 153 first-year students nationwide in 2002. The enrollment has much increased, with 591 students in 2012 and 673 students in 2013; see the graph below. In higher professional education, student numbers in industrial mathematics are also on the rise. The inflow of 2013 was 295, which more than doubles the inflow of 2008. Possible explanations for the steady growth of inflow of mathematics students are the increased societal appreciation for the mathematical sciences and the more positive perception of mathematics among high school students, which may be explained by the introduction of the *Wiskunde D* module in secondary education and the recent outreach activities by the mathematics community.

## Student enrollment and staff size over time



*high quality of education and research*

The increased enrollment has led to a healthy student population in the bachelor and master mathematics programs. The bundling of several of these programs has reinforced the high quality of Dutch mathematics education. The clusters have provided focus and coherence in Dutch mathematics research and helped to maintain its high quality. Bachelor programs give extra attention to excellent students by means of honor programs and double-degree bachelor programs, and to weaker students in the form of tutor classes. Mathematics students receive much individual guidance in individual projects and thesis advising, in the style of the tried and tested master-apprentice model.
Educational and research evaluation committees traditionally judge Dutch mathematics groups to be very good or excellent at an international level. This was confirmed by the cluster evaluation of 2011 [NWO 2011a], in which an international committee assessed the performance of the three initial clusters in research, teaching and organization to be *excellent*, while the research quality of DIAMANT was considered to be even *exemplary*.

*external orientation, universal problem-solving capability*

Mathematicians are problem solvers. The mathematical sciences have age-old roots in engineering and physics, and their links to the sciences, to technology and to practice are getting stronger every day. The focus on the universal structures and abstract methodologies of problem solving allows mathematical ideas to be applied across the whole spectrum of applications. Dutch mathematicians in academia engage in a wide range of collaborations with industry. Additionally, there are strong collaborations with other areas of science, especially life sciences, earth sciences and ecology.

*inaccessibility of subject, invisibility of results*

Mathematicians are concerned with abstract ideas. They have developed logical constructions and symbolic language in order to be able to work efficiently with abstractions, to express ideas precisely and succinctly, and to perform formal calculations with confidence in the results. All of this makes mathematics hard to learn and hard to communicate to non-mathematicians. The results of mathematics are powerful but often hidden within the application. Hence, many people remain unaware of the profound influence of mathematics on their daily lives.
Also, due to the steep learning curve of mathematics, the time required to obtain output from mathematics research may be quite long. PhD students need several years to

produce deliverables, and research results may have no validation horizon within sight. This is often a deterrent to collaborations with industry. Long-term thinking is frequently at odds with industrial planning.

Many disciplines which properly belong to the mathematical sciences – statistics, operations research, scientific computing and control theory, to name a few – are so highly integrated in other fields that these "lay claim" to mathematical results when convenient. As a result, mathematicians may be neglected or bypassed in initiatives of a mathematical nature. The integration, however, is an inevitable development, which will eventually offer opportunities to the mathematical sciences in the broad sense. The top sector policy of the Dutch government is an evident example; see Section 4.3. *See Recommendation 6.*

*low participation of women and minorities*

The percentage of female undergraduates in Dutch mathematics studies is higher than in engineering disciplines, but the percentage of ethnic minorities is low, and the participation of women and minorities drops quickly with each step up the academic career ladder. We do not succeed in mobilizing the talent available in those groups. This issue is not specific to our field and there is no quick solution. The recruitment and retention of women and minorities needs continuing attention. Targeted hires and the involvement of women and minorities in visible activities and responsible positions may help. *See Recommendation 5.*

*lack of breadth in PhD education*

For PhD students in mathematics in the Netherlands, there are too few mandatory advanced education programs. However, since many developments occur at the interface of subdisciplines, mathematics research requires a broad knowledge as well as general skills. The mathematics clusters, in cooperation with WONDER, the Dutch Research School in Mathematics, are in a position to play an initiating role in the development of high-quality national graduate education programs. *See Recommendation 2.*

*broadening reach and impact*

The ongoing mathematization of the sciences, industry and society presents great opportunities for the mathematical sciences. Computational science is a terrain in which mathematical and application scientists naturally meet. As the sciences become more mathematical, they will increasingly challenge mathematicians to improve their methodologies. At the same time, new applications, for example in the life sciences and the social sciences, are often not readily cast in known mathematical structures. This is an exciting situation that offers big opportunities, because it requires the development of truly new mathematics.

The expansion of the mathematical sciences underlines the more prominent role that they should play in STEM education. We need to design new models of teaching mathematics, and interdisciplinary tracks and programs calibrated to the needs of society and industry.

For the mathematics students, a diversification of their education is quite feasible in view of the larger student population and the national collaboration. A study by SIAM [SIAM 2012] points out that mathematicians employed by industry need a broad knowledge of the mathematical sciences, computational skills, and expertise in at least one application domain. In addition, general skills are required including an interdisciplinary attitude, an ability to communicate, and a sense of business.

Also the teaching of mathematics to students in subjects that increasingly depend upon the mathematical sciences needs attention. At the Dutch universities of technology, the mathematics staff is highly involved in "service teaching". At some of the other universities, it is hampered by inadequate financial accounting models. *See Recommendation 3.*

*globalization and information revolution*

Globalization and the societal information revolution are changing the way we do research and teach. They are stimulating real-time international collaboration, and make scientific literature widely available and rapidly searchable. The means of globalization, that is, the internet, the web and social media, and the associated data analysis are objects of mathematical study themselves. The mathematics of information, including

network theory, big data research, operations research and game theory, has the opportunity to guide the transitions to smart power grids and to automated logistics and transport.
The huge research investments in emerging economies such as China, India and Brazil lead to a substantial growth of talented and motivated students that wish to study abroad. The fast-growing internet creates opportunities for online teaching. The massive open online courses that are being offered in the United States may introduce the “flipped classroom", where students follow lectures at home and problem-solving sessions in class. Dutch academia is well positioned to benefit from these trends: English is its second language and education is of a high level and inexpensive. It requires swift action on online teaching and also a reassessment of the scholarship systems for foreign students to study in our country.

*small volume, high and increasing workload*

It is an essential aspect of academia that teachers in bachelor and master programs are also active researchers. Research and teaching strengthen and enrich each other. To safeguard the quality of both, care has to be taken that the workload is reasonable and in balance. The workload of mathematics staff members has substantially increased, however. On the research side this is caused by the growing pressure to obtain external funding. On the teaching side it is due to the growing student numbers since 2003 without a matching increase in staff; see the graph above. Even the investment of the clusters in 24 new positions in 2005 has not led to a growth in permanent staff. In the mathematical sciences, the productivity of research, the quality of teaching and the individual guidance of students are at stake. *See Recommendation 1.*
The small size of the mathematics departments is an obstacle to a steady inflow of talented staff. The creation of individual career paths may help in attracting and keeping mathematical talent. *See Recommendation 5.*

*funding of fundamental research under pressure*

Both the European Commission [EC 2013] and the KNAW [KNAW 2013] offer strong pleas in favor of safeguarding budgets for fundamental research. Nevertheless, the funding opportunities for fundamental research are under pressure. The “free competition” of the Physical Sciences Division of NWO was, for a long time, its main instrument for small-scale funding in the form of individual PhD projects. At present, junior researchers can still apply for such projects, without having to compete with senior researchers who have a proven track record; for the latter, there are collaborative schemes. On the whole, funds are limited, and established researchers cannot apply for individual projects. They can consolidate their research into larger initiatives, or apply to European programs. Despite recent successes, the European Research Council (ERC) is underused by Dutch mathematicians as a source of research funding. *See Recommmendation 6.*

## Gravitation project “Networks”

**Transportation, communication and energy networks form an essential part of our society’s critical infrastructure. They provide formidable research challenges in mathematics and computer science: they fluctuate wildly, are highly volatile, and lack central control. The Dutch government recently awarded 23 M€ to the Gravitation project “Networks” for the coming ten years. Researchers at the University of Amsterdam, Eindhoven University of Technology, Centrum Wiskunde & Informatica and Leiden University propose to combine stochastic modeling with algorithmic tools to describe, control, optimize and predict network processes at a fundamental level, and aim to develop a universal theory of intelligent and self-organizing networks.**

## 4.2.

# Primary and Secondary Education

*high quality of education*

Mathematics is a core subject in Dutch primary and secondary education. Mathematical competence is a basic condition for participating in a knowledge-based society. At a more advanced level, it is a prerequisite for much of higher education. Mathematics as a school subject also plays a key role in awakening curiosity and interest in questions of a scientific and technological nature. A strong mathematical foundation is critical to the success of an advanced economy.

*math competence in primary and secondary education falling back*

Dutch education is traditionally in the international top league. Recent PISA and TIMSS surveys [OECD 2010, 2013, IEA 2012] still give high scores to the mathematical competence of Dutch school pupils, but show that it is slowly falling back. The instruction time on mathematics in secondary education is small in comparison to neighboring countries. In the long run, these are threats to the level of students in higher education, to the functioning of our society, and to the competitive position of our country.

*erosion of teacher education*

A study by the KNAW [KNAW 2009] showed that the decrease of mathematical competence in primary schools is mainly due to an erosion of the teacher education. The quality of the teacher is the predominant factor in the quality of education. The Dutch teacher training colleges have been suffering, for several decades, from a low quality level of intake and an undue emphasis on general pedagogical knowledge at the cost of subject knowledge in, e.g., language and mathematics. In addition, post-initial training of teachers has been fading away.

The teacher training colleges are presently going through a process of restoring the balance in the curriculum and raising the subject mastery level. This is a complex and long-term process, embraced by the colleges and stimulated by the government. Another hopeful sign is that universities have started academic teacher programs, newly designed tracks for educating new types of primary school teachers.

*no tracks fostering excellence in primary schools*

International surveys show that Dutch primary education pays much attention to slow learners but very little to more gifted pupils. There is a need for programs that foster excellence and stimulate talented pupils, who may be expected to contribute to innovation. *See Recommendation 5.*

*role of university in teacher education*

In secondary education, the teachers are the ambassadors of thought and learning.

At present, the large majority of high school teachers are educated at teacher academies, not at universities. The universities should view it as their mission to resume their role in educating first-level high school teachers. This will require a reshaping of the current curriculum. After a bachelor degree in, e.g., mathematics, the master program should be structured in a flexible and attractive way. The Association of Netherlands Universities has presented an action plan in this regard [VSNU 2013]. *See Recommendation 4.*

Both secondary and university education would profit from creating dual appointments between them. It could grade up the quality of teaching and would facilitate the transition of students to university. *See Recommendation 4.*

*Teachers are the ambassadors of thought and learning.*

Mathematics teaching in secondary education is now organized in the form of four tracks, *Wiskunde A-D*, with the purpose to offer students of each profile a mathematics education that fits their needs and abilities. *Wiskunde D* is an elective for science-oriented students. It is offered by about seventy-five percent of pre-university high schools (*athenaeum* and *gymnasium*) and followed by on average five or six students per school.

It contributes to fostering excellence and is believed to contribute to the growing inflow of mathematics students at universities. Unfortunately, the number of schools offering *Wiskunde D* appears to be declining. *See Recommendation 5.*

The improved attitude of students is another positive development. We elaborate on this in Section 4.3.

*improved attitude of students*

Finally, we mention the masterplan *Naar 4 op de 10* (*Towards 4 out of 10*), which was recently launched by the top sectors of the Dutch economy (see Section 4.3) in cooperation with the Platform Bèta Techniek. It aims at increasing the percentage of high school students opting for study in STEM subjects from the present twenty-five to forty in 2025.

## 4.3.

## Society and Innovation

*broadening reach and impact*

The mathematical sciences have become a catalyst for change and a major factor for growth. In its study for the UK, Deloitte [Deloitte 2014] distinguishes three types of contributions of the mathematical sciences to society and innovation: making sense of data and better understanding the world, safeguarding society, and creating robust forecasts and optimized processes. SIAM [SIAM 2012] mentions three new application domains that have become prominent in the 21st century: life sciences, medicine, and healthcare; financial engineering; and services, which benefit from analytics and optimization as manufacturing did in the past.

Several developments are causing this broadening reach and impact of the mathematical sciences. The trend to a global economy and a knowledge society is placing information technologies at the forefront; these increasingly depend on research driven by mathematics. The efficient development of novel products and technologies, essential for a world with an aging population and limited resources, also depends upon results from the sciences and mathematics. In almost all industries, mathematics opens the way to virtual experiments and to the simulation and analysis of multiple scenarios.

Accordingly, Dutch graduates in the mathematical sciences have excellent employment possibilities. Many of them are taken up by business and industry. Mathematicians are viewed as multifunctional, logical thinkers.

*The mathematical sciences have become a catalyst for change and a major factor for growth.*

*high quality of applications*

The high quality of the mathematics contributions to Dutch society is well recognized. As a striking example, we mention that the Franz Edelman Award of the Institute for Operations Research and the Management Sciences “for outstanding examples of operations research, management science, and advanced analytics in practice” went to the Netherlands three times in the past six years: in 2008 to the Netherlands Railways, in 2012 to TNT Express, in 2013 to the Dutch Delta Program Commissioner. Each of these projects was carried out in close cooperation with Dutch academic groups.

*external orientation, universal problem-solving capability*

The external orientation of Dutch mathematics, and of the Dutch sciences in general, is also evident from the frequent appointment of leading researchers from industry as part-time professors at universities, which leads to strong links between industrial challenges and academic research. Another direct link is the successful carousel of the Study Groups *Mathematics with Industry*, which have been organized every year since 1998 at a different institute, and in which fifty to eighty mathematicians spend a week investigating five or six problems from industry.

*outreach activities*

*public at large educated and fascinated*

*improved attitude of students*

Intensive outreach activities have recently contributed to improving the perception of the mathematical sciences for a general audience and among students. The activities fell on fertile ground, fed by the high educational level of the Dutch population and its fascination with science issues. Thanks to the elective *Wiskunde D* module in pre-university secondary education, the mathematical olympiads, master classes and summer schools, an increasing number of high school students finds math “cool”. For them, in comparison to some twenty or thirty years ago, the culture of mediocrity is gradually giving way to a culture of knowledge, learning and excellence. The society views students that wish to stand out in a positive way, and instead of being shunned, they feature more prominently in society. *See Recommendation 7.*

*funding by industry and other sources*

In 2010 the Dutch government decided to direct its budgets for research and innovation towards the needs of nine top sectors of the Dutch economy. Nowadays a major part of the Dutch research budget is allocated to the top sector policy, which also heavily relies on funding from industry. The societal challenges embodied in this policy present opportunities for researchers to contribute to some of the biggest problems confronting the country and the world, such as energy supply, health, and water management. Mathematicians need to embrace this opportunity and invest time and effort in forging strategic alliances. It is an unfortunate fact that the positioning of the mathematical sciences in relation to the top sectors is less than ideal. Other disciplines are directly related to specific top sectors, whereas the mathematical sciences tend to contribute indirectly to all of them, via other scientific and engineering disciplines. Mathematical results are often hidden deep away in applications, and the incubation time from research to results is long. In addition, mathematicians are used to thinking in terms of small individual projects, instead of taking charge of large transdisciplinary initiatives. A successful participation in the top sector policy and, in general, in public-private programs requires a different mindset. *See Recommendation 6.*

For several societal challenges, funding may be sought through European instruments. An example is climate change and its consequences. In 2014 the European Framework Program *Horizons 2020* will commence.

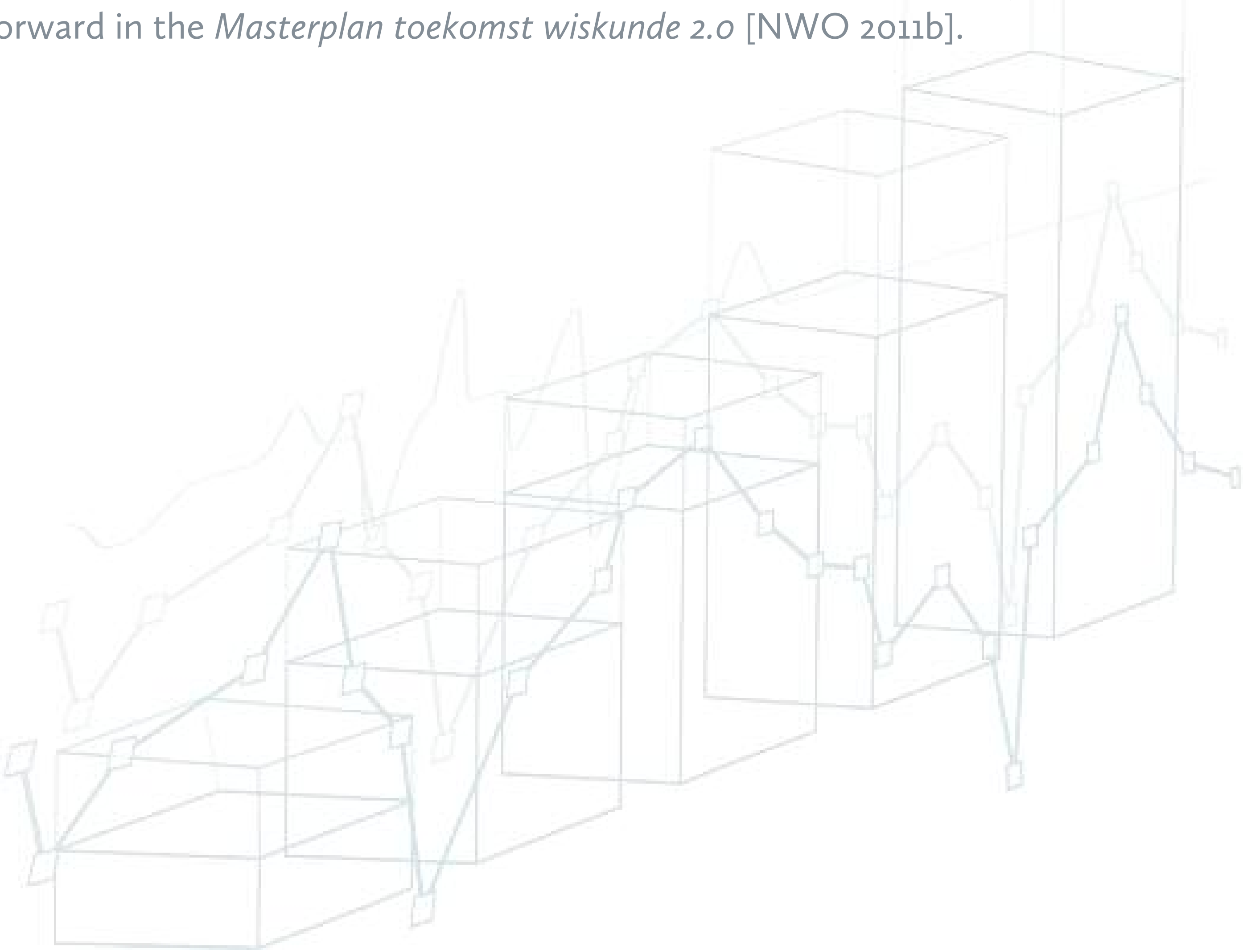
*funding of research top-down and demand-driven*

The governmental focus on steering research in specific societal domains marginalizes the role of the scientific community in setting the research agenda and may, in the long run, endanger all of fundamental science. In this context, the KNAW [KNAW 2013] pleads in favor of a long-term vision of science and recommends restoring budgets for fundamental research. We have argued above that a healthy core is the primary source for the innovations that the mathematical sciences bring to other disciplines and to society in the form of creative, efficient and reliable solutions. Without room for fundamental exploration, creative problem solving will dry up. *See Recommendation 8.*

# 5

# Conclusions and Recommendations

Our conclusions and recommendations naturally emerge from the SWOT analysis in the previous chapter. We will focus on the recommendations that should have an effect in the medium term, say, by 2025. They are directed towards policymakers in government, granting organizations, educational councils, universities and schools, but also towards the mathematical scientists themselves. Some of our recommendations were already put forward in the *Masterplan toekomst wiskunde 2.0* [NWO 2011b].



### 1 – Balance workload

The decline in mathematics student numbers that started in 1989 has led to a reduction in mathematics staff by forty percent overall. The growth of student numbers since 2002 is a positive development for the mathematical sciences and for society as a whole. However, it has not been matched by an increase in staff, in spite of appointments from cluster funds. The quality of research, teaching and student guidance is at stake.
**Recommendation.** *Expand the mathematics staff in academia by at least fifty percent, in order to balance the primary tasks of teaching and research, and to safeguard their quality.*

### 2 – Consolidate dynamic clusters

The clusters have proven to be very beneficial to the Dutch mathematical community. They provide focus, ensure critical mass in four broad areas, help in maintaining quality, and encourage interaction. They represent a bundling of forces that is necessary in order to compete on the highest level. Their funding, however, has been ad hoc and short-term.
**Recommendation.** *Provide structural long-term funding to the mathematics clusters, subject to periodic evaluations. The clusters themselves should prove their vitality by evolving in response to developments of the discipline and by being flexible in their composition. They should initiate the development of high-quality national graduate education programs.*

*The clusters have proven to be very beneficial to the Dutch mathematical community.*

### 3 – Reform academic mathematics teaching.

The increasing reach of the mathematical sciences leads to a higher demand for mathematical scientists across a broad spectrum of technological and societal occupations, with a greater diversity in mathematical skills and more emphasis on general skills.
**Recommendation.** *In the light of the increased and more diverse demand for mathematical scientists, academia needs to reassess the mathematics education. The mathematics staff should be highly involved in the service teaching of mathematics to students in other subjects.*

### 4 – Secondary education and academia

The primary task of secondary education is preparing students for the choices that lay ahead of them and stimulating them to optimally contribute to society. From this perspective, it is essential to raise the academic level of high school teachers and to forge closer links between secondary education and academia.
**Recommendation.** *The universities need to resume their role in educating high school teachers, by restructuring the curriculum and making it flexible and attractive. The universities and the Council for Secondary Education should develop a scheme for creating and stimulating dual appointments between high schools and universities.*

### 5 – Foster talent

It is important to recognize mathematical talent early and to provide gifted pupils and students with additional stimuli. In primary education, insufficient attention is given to gifted students. In secondary education, the organization of mathematics teaching in four tracks seems to work well; it is too early to judge the effect of the government program *Naar 4 op de 10*. In academia, the small size of the mathematics departments hampers a steady inflow of talented staff; the participation of women and minorities, especially after the bachelor, is still too low.
**Recommendation.** *The Council for Primary Education should develop a program for introducing tracks and tools that foster excellence and stimulate talented pupils. Pre-university high schools should be encouraged to maintain the* Wiskunde D *elective. The universities should create career paths for attracting and keeping mathematical talent; especially the recruitment and retention of women and minorities needs continuing attention and monitoring.*

## 6 – Funding in transition

The Dutch landscape for research funding is in transition. Due to the new policy of the Physical Sciences Division of NWO and the government's focus on funding research through nine top sectors of the economy, it has become harder for mathematicians to obtain small-scale individual projects. The ERC is, until now, underused as a source of mathematics research funding.
**Recommendation.** *The budget of NWO for funding fundamental research without immediate economic benefits should be expanded. At the same time, mathematicians need to take more advantage of European programs and embrace the many new opportunities offered by emerging areas of the mathematical sciences, by larger cooperative initiatives, and by strategic alliances with industry. Through its innovation committee, PWN should take charge of promising thematic opportunities.*

## 7 – Reaching out

Intensive outreach activities have recently contributed to improving the perception of the mathematical sciences in general and among students. Mathematicians have become aware of the need to communicate the pride of their discipline, the fascination for discovery, and the thrill of successful applications.
**Recommendation.** *Mathematicians should continue and expand their outreach activities. PWN should help by professionalizing communication.*

## 8 – Vision of science

The governmental policy of steering research as a function of present economic demand may, in the long run, endanger fundamental science. Our plea for a more balanced long-term vision of science transcends the interests of the mathematical sciences and is shared by the Royal Netherlands Academy of Arts and Sciences [KNAW 2013].
**Recommendation.** *The Dutch government needs to formulate a long-term vision of science, taking the needs and opportunities of the Dutch society and economy into account and recognizing that fundamental research is the principal source for innovation in the long run. The Dutch academic community should actively engage in this process.*

Our final and main recommendation aims at realizing the above, more specific, recommendations. It again has a dual nature: it is primarily directed towards the government, but also requires concrete efforts by the mathematical community in taking initiatives and securing additional structural funding.

*Our society needs the mathematical sciences, and the mathematical sciences need to be enabled to play their role.*

## 9 – Implementation plan

The report by Deloitte [Deloitte 2014] underlines the urgency of increased attention of policymakers in the Netherlands for the mathematical sciences. Our society needs the mathematical sciences, and the mathematical sciences need to be enabled to play their role.
**Recommendation.** *The Minister of Education, Culture and Science should establish an implementation committee with the task of drafting a detailed implementation plan for the Dutch mathematical sciences in the medium term. The committee will address the needs of the mathematical sciences in the Netherlands along the lines of the present report, with the purpose of safeguarding fundamental research and realizing the above recommendations. It will formulate action lines for the mathematical scientists in higher education and research, in primary and secondary education, and in society and innovation, and set measurable milestones for student inflow, student throughput, educational innovation, university-trained school teachers, share in European funding, gender, and outreach. This will allow the Minister to monitor the effectiveness of additional funds and to make these funds structural after proven success.*

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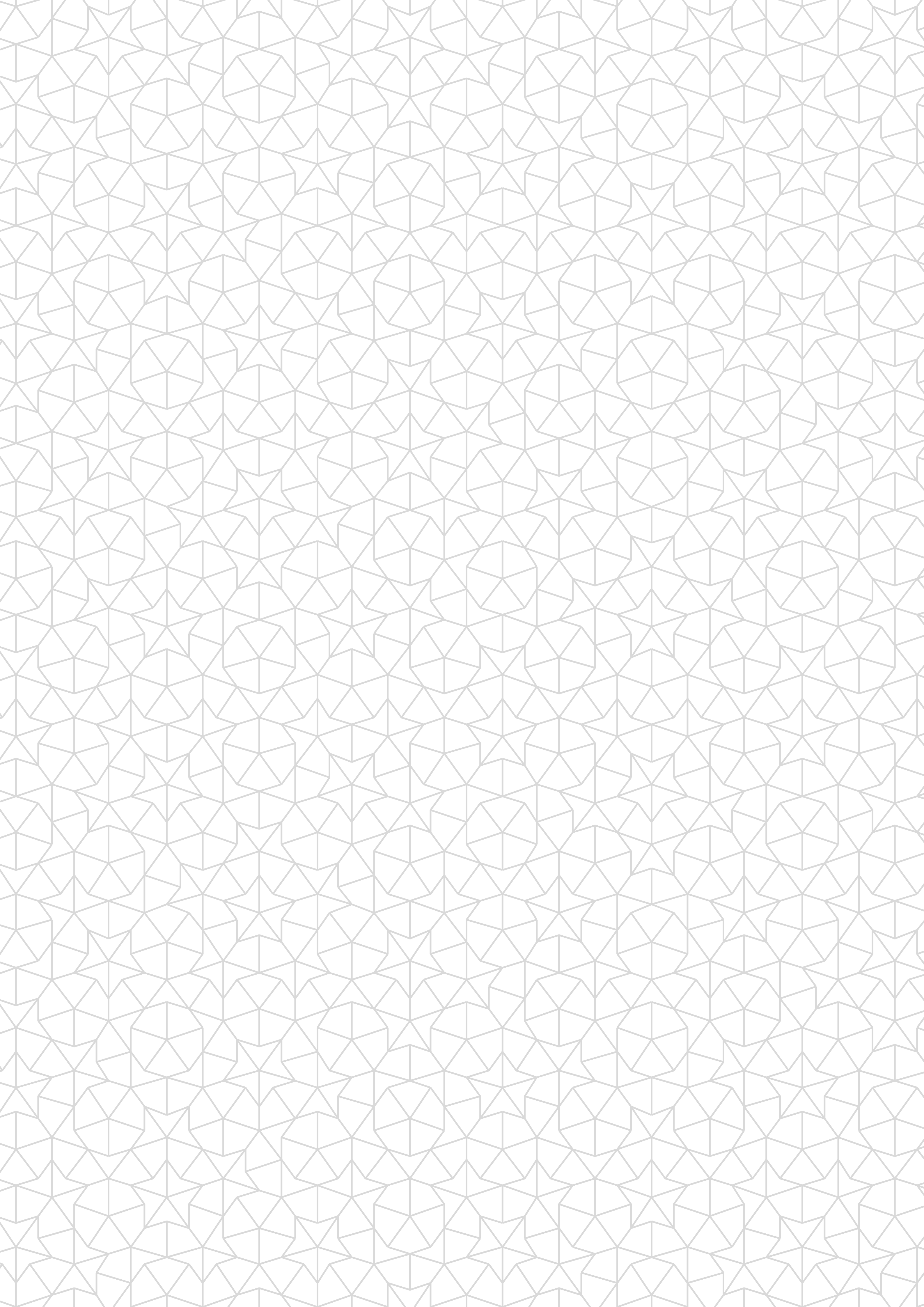

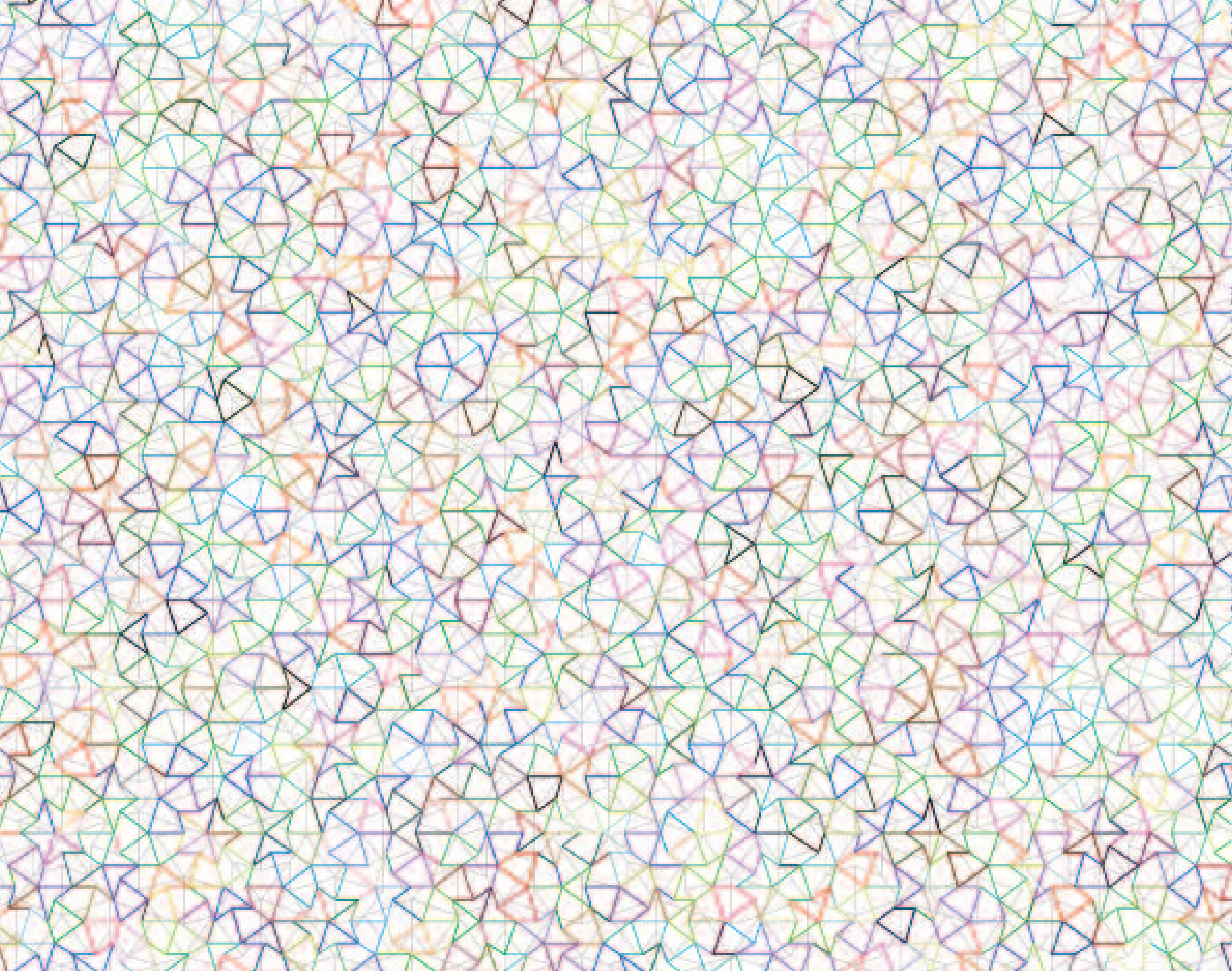

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