In[1]:=
```
Clear["Global`*"];
f[z_] := (-7 z + 1)/((z - 1) (z - 7));
```

In[3]:=
```
Residue[f[z], {z, 1}]
```

Out[3]= 1

In[4]:=
```
Residue[f[z], {z, 7}]
```

Out[4]= -8

In[5]:=
```
g[z_] := (2 + z)/(1 - Sin[z]);
Residue[g[z], {z, π/2}]
```

Out[6]= 2

In[7]:=
```
h[z] := z/((z - Sin[z]) (Cosh[z] - Cos[z]));
Residue[h[z], {z, 0}]
```

Out[8]= 0

In[9]:=
```
Integrate[Cos[m*x]/(1 + x^2),
 {x, -∞, ∞}]
```

Out[9]= $\text{If}\left[m \in \text{Reals}, e^{-\text{Abs}[m]}\,\pi, \text{Integrate}\left[\frac{\text{Cos}[m\,x]}{1+x^2}, \{x, -\infty, \infty\}, \text{Assumptions} \to \text{Im}[m] < 0 \,||\, \text{Im}[m] > 0\right]\right]$

In[10]:=
```
-2 π * Im[Residue[Exp[I*m*z]/(1 + z^2), {z, I}]]
```

Out[10]= $e^{-\text{Re}[m]}\,\pi\,\text{Cos}[\text{Im}[m]]$

In[11]:=
```
cosof[z_] := (z + 1/z)/2;
integrand = Simplify[-I/(z*(8*(cosof[z])^2 + 1))]
```

Out[12]= $-\frac{i\,z}{2 + 5\,z^2 + 2\,z^4}$

In[13]:=
```
poly = Denominator[integrand];
z /. Solve[poly == 0, z]
```

Out[14]= $\left\{-\frac{i}{\sqrt{2}}, \frac{i}{\sqrt{2}}, -i\sqrt{2}, i\sqrt{2}\right\}$

In[15]:=
```
res1 = Simplify[Limit[(z + I/Sqrt[2]) integrand, z -> -I/Sqrt[2]]]
```

Out[15]= $-\frac{i}{6}$

In[16]:=
```
res2 = Simplify[Limit[(z - I/Sqrt[2]) integrand,
   z -> I/Sqrt[2]]]
```

Out[16]= $-\frac{i}{6}$

In[17]:=
```
2 π I (res1 + res2)
```

Out[17]= $\frac{2\pi}{3}$

In[18]:=
```
Integrate[1/(8*(Cos[x])^2 + 1), {x, 0, 2 π}]
```

Out[18]= $\frac{2\pi}{3}$

In[19]:=
```
Integrate[(x - Sin[x])/x^3, {x, 0, ∞}]
```

Out[19]= $\frac{\pi}{4}$

In[20]:=
```
Integrate[(x - Sin[x])/x^3, {x, -∞, ∞}]
```

Out[20]= $\frac{\pi}{2}$

In[21]:=
```
Series[(I z - e^(I z))/z^3, {z, 0, 6}]
```

Out[21]=
$$-\frac{1}{z^3}+\frac{1}{2\,z}+\frac{i}{6}-\frac{z}{24}-\frac{i\,z^2}{120}+\frac{z^3}{720}+\frac{i\,z^4}{5040}-\frac{z^5}{40\,320}-\frac{i\,z^6}{362\,880}+O[z]^7$$

In[22]:=
```
Series[(I z - e^(I z) + 1)/z^3, {z, 0, 3}]
```

Out[22]=
$$\frac{1}{2\,z}+\frac{i}{6}-\frac{z}{24}-\frac{i\,z^2}{120}+\frac{z^3}{720}+O[z]^4$$

In[23]:=
```
graph = {AbsoluteThickness[3], AbsolutePointSize[6],
   Circle[{0, 0}, 1, {0, π}], Circle[{0, 0}, 1/10, {0, π}],
   Line[{{-1, 0}, {-1/10, 0}}], Line[{{1, 0}, {1/10, 0}}],
   Point[{0, 0}], Text["C₁", {-0.5, -0.1},
    BaseStyle → {FontSize → 15}],
   Text["C₂", {0.15, 0.15}, BaseStyle → {FontSize → 15}],
   Text["C₃", {0.5, -0.1}, BaseStyle → {FontSize → 15}],
   Text["C₄", {0.5, 0.75}, BaseStyle → {FontSize → 15}]};
```

In[24]:=
```
Show[Graphics[graph, AspectRatio → Automatic, Axes → True,
  Ticks → None, PlotRange → {{-1.2, 1.2}, {-0.3, 1.2}}]]
```

Out[24]=