Coherence With and Without Unique Normal Forms

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One may define a structure on a category to be a two-dimensional system of generators and relations. In the case where all of the structuring functors are covariant in all of their arguments, Kelly [3] showed how to generate a doctrine, $\mathcal{D}$, whose algebras are categories endowed with the free such structure. Viewing a coherence problem as being concerned with the commutativity of diagrams in the initial $\mathcal{D}$-algebra on a discrete category, one may delineate two related coherence questions: (1) Do all diagrams commute?, and (2) Is there a decision procedure that determines whether a given diagram commutes?

We recast Kelly's construction in the framework of term rewriting theory, where the initial $\mathcal{D}$-algebra on a discrete category becomes a term rewriting system modulo a two-dimensional congruence. Within this framework, it is possible to resolve the above coherence questions in terms of the underlying rewrite system. We shall briefly discuss a resolution to (1) for structures described by rewriting systems with unique normal forms. Such systems lend themselves to arguments involving induction on the length of the longest reduction to a normal form. The solution we provide can be seen as a natural generalisation of Laplaza's proof of coherence for distributive categories [4] and a close relation of Johnson's general coherence theorem for a class of n-categorical pasting schemes [2]. Following this, we describe an alternative approach via planar subdivisions, which applies also in the case where the system does not enjoy unique normal forms, and obtain sufficient conditions for the above two questions to be resolved in the affirmative. An important example of a system without unique normal forms is provided by iterated monoidal categories [1] and we demonstrate how our results can be used to give a conceptual approach to the coherence problem for these structures.

References

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