Generalized higher Hopf formulae and
a long Stallings-Stammbach exact sequence

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Using categorical Galois structures [7] of *higher extensions* and *higher central extensions*, Brown and Ellis's higher Hopf formulae [1] for the homology of a group can be defined in any semi-abelian category $\mathcal{A}$ [8], relative to a choice of Birkhoff subcategory $\mathcal{B}$ of $\mathcal{A}$. We establish a long exact sequence of higher Hopf formulae, which extends the well-known Stallings-Stammbach five term exact sequence in the case $\mathcal{A} = \mathsf{Gp}$ is the variety of groups and $\mathcal{B} = \mathsf{Ab}$ is the subvariety of abelian groups. We apply our results to the situation where $\mathcal{A} = \mathsf{PXMod}$ is the category of precrossed modules and $\mathcal{B} = \mathsf{XMod}$ the category of crossed modules.

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