

# Model structures on the category of double categories

Tom Fiore \*

When are two categories the same? One possible notion of sameness is equivalence of categories: two categories are the same if there is a fully faithful essentially surjective functor between them. A coarser possibility is to consider two categories the same if there is a functor between them whose nerve is a weak homotopy equivalence of topological spaces. As is well known, these distinct notions of weak equivalence between categories have been encoded in model structures by [2] and [5].

One can ask the same question for double categories: when are two double categories the same? We collate the various notions of weak equivalence into model structures on the category of double categories. This complements recent work on model structures on **BiCat** and **2-Cat** in [3], [4], and [6] as well as the work [1] on model structures on internal categories.

## REFERENCES

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