Towards Bicategorical Quantum Mechanics the calculus of quantaloidal enrichment and its applicability to Quantum Mechanics

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The application of conceptual and formal tools from category theory to phycics in general and to Quantum Mechanics (QM) in particular is a relatively new yet dynamically developing field of research, attracting the interest of both categorists and theoretical physicists. The distinctive and at the same time puzzling features of quantum mechanical systems like *entanglement*, *non-locality* and *indeterminism* suggest that a categorical approach to QM should primarily afford the grasping and formalisation of a notion of *quantum varying structure* representing the evolution of quantum systems and processes in a profound manner.

Understandably enough, the first attempts for such a grasping were implementations of the *topos-theoretical* machinery, since topoi, in particular topoi of *(pre)sheaves*, are the almost canonical example of varying categorical structure. However, not only is the intuitionistic internal logic of topoi rather incompatible with the distinctively non-commutative "logic" of QM, but also (pre)sheaves themselves, standardly being a kind of *varying sets*, can only provide a restricted representation of quantum mechanical variation. The latter seems to necessitate a more involved and implicated notion of "quantum sheaf" and "space" than the topos-theoretic one. Indeed, both the physics and the formalism of QM indicate that quantum mechanical systems and processes should be considered as *structured* objects, in other words *categories themselves*, having a non-trivial internal structure, living accordingly in a non-trivial ambient category too.

Category theory does provide the tools to develop a theory of such non-trivial objects-categories and their ambient environment. The relative "cost" in achieving this is that someone has to ascend the ladder of categorical complexity, employing not ordinary categories but categories of a higher order, known also as 2-categories and *bicategories*. To be precise, quantum systems *qua* objects-categories should be considered *enriched* categories. In fact, not only their ambient category but, even more importantly, the background category for the enrichment as well should be 2-categories. Indeed, it is enriching upon such a 2-category or a bicategory that the internal structure and the "dynamics" of such quantum objects can be properly probed and understood. For example, the 2-dimensional generalisation of ordinary sheaves (eg. in the form of stacks, see [3]) is implemented in the context of *variable categories* and a notion of *processes between processes*, as explained for instance in [1], does naturally involve bicategories (the special place of bicategories in the categorical ladder is nicely commented for instance in [2]).

At first sight enriching over a bicategory looks quite a taunting task. However, there is a particular kind of bicategories which is very convenient to start with, namely quantaloids. Intuitively speaking, a quantaloid is a category whose homobjects are not just sets but complete suplattices. Quantaloids provide thus a handy calculus where 2-cells are just inequalities and all diagrams involving them commute by definition (for the relevant theory someone may follow for instance the series of papers by Isar Stubbe, eg. [4]).

Enriching over a quantaloid $\mathcal{Q}$ gives rise to two categorical structures, the quantaloid $\mathsf{Dist}(\mathcal{Q})$ of $\mathcal{Q}$-enriched categories and the *distributors* (also known as profunctoes or bimodules) between them and the locally partially ordered 2-category $\mathsf{Cat}(\mathcal{Q})$ of $\mathcal{Q}$-enriched categories and *functors* between them.

Our first aim towards what may be called **Bicategorical Quantum Mechanics**, is to develop and interpet the calculus of quantaloidal enrichment in order to recover the main elements of the standard QM and its formalism. Indeed, interpeting $\mathcal{Q}$-enriched categories as the quantum mechanical objects of our discourse, homarrows as *transition* amplitudes, the so-called presheaves on them as generalised "states", and functors as generalised "operators", leads to a considerable transcription of the main body of the formalism of ordinary QM. Even further, by means of a tentative analysis of the embedding of the quantum system into its free cocompletion (what we call the *space of dynamics* of the system) and to its *Cauchy* completion, we entertain the idea of a radical re-interpretation of quantum mechanical *measurement* as a *universal* construction in this bicategorical context.

Although still in the beginning of developing such an approach, we may argue that it has the potential to provide us with valuable insights to a consistent and intuitive understanding of the renowned conceptual intricacies of QM.

References

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