The theory of glueing things on

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The notion of weak factorisation system provides a convenient framework for the discussion of higher-dimensional structure that a mere category may possess. Yet its definition deals not in universal properties, but in *weakly* universal properties, rendering inapplicable much of the machinery that category theory could otherwise bring to bear.

This deficiency may be overcome if we move from weakly universal properties to algebraic structure. The *natural weak factorisation systems* (n.w.f.s.'s) of [2] effect such a transition: they are given by a monad-comonad pair satisfying various equational laws, which, whilst retaining the expressive power of weak factorisation systems, gain many of the pleasant properties of the theory of monads.

In this talk, we study one instance of this. The *cofibrantly generated* n.w.f.s.'s of [1] provide a notion of n.w.f.s. generated by a signature of basic operations: these operations being specified by a regulus of maps which we wish to belong to the left class of our n.w.f.s. This construction is "free", providing the syntax side of a syntax/semantics adjunction; and may be viewed on the one hand, as a suitably refined free monad construction of the sort studied by Barr, Kelly, Wolff, *et al*; and on the other, as an improved version of the "small object argument" deployed in the construction of plain w.f.s.'s.

Cofibrantly generated n.w.f.s.'s frequently give rise to "weak morphism classifiers"; and we end with some examples of this.

REFERENCES

[1] Richard Garner, *Cofibrantly generated natural weak factorisation systems*, arXiv preprint (2007), math.CT/0702290.

[2] Marco Grandis and Walter Tholen, *Natural weak factorization systems*, Archivum Mathematicum 42 (2006) 397–408.