

Cospans and weak cubical categories in Algebraic Topology

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Topological cospans and their composition, by pushout, appear in the theories of tangles, ribbons, cobordism, etc. Various algebraic invariants have been introduced for their study. We show how (co)homotopy and (co)homology functors can be used to provide new algebraic invariants, in the less usual form of spans, or cospans, or relations between sets, or groups, or abelian groups. Under suitable restrictions, such functors can be ‘linearised’ (with values in finite dimensional vector spaces), and give – for instance – topological quantum field theories for manifolds and cobordisms.

We end with introducing n -cubical cospans $\Lambda^n \rightarrow X$ (in a category X with pushouts), where Λ is the ‘formal cospan’ category. These diagrams form a ‘weak cubical category’, i.e. a cubical set with compositions in all directions, which behave categorically in a weak sense, up to suitable comparisons. This structure will be used for the higher dimensional versions of the previous invariants.