Comprehending structure types and stuff types

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Graph comprehension provides a means of reconciling two different views of labeled transition systems: as (faithful) graph morphisms into the graph, or better category, of labels, and as graph morphisms from the later into the bicategory **Spn** of spans (**Rel** of relations) over sets. It specializes to a number of known equivalences, for instance between fibrations into a category B and contravariant pseudo-functors from B into the bicategory **Cat**, or between functions into a set C and C-indexed families of sets.

Species of structures, on the other hand, were introduced 1981 by Joyal in order to "categorify" combinatorics. They can be viewed as functors from the groupoid E of finite sets into itself, or into the category of finite sets. Viewed from a different angle, they correspond to so-called "structure types", namely faithful functors from some groupoid into E. In 2001 the later concept was generalized to "stuff types" by Baez and Dolan by dropping the assumption of faithfulness.

Building upon the categorical analysis of stuff types started by Simon Byrne in 2005, we show how stuff types fit into the framework of graph comprehension, namely by specializing to symmetric graphs, respectively, symmetric spans. The corresponding transition systems account for reversible computations, like those that can be performed by a quantum computer. Furthermore, various types of morphisms between labeled transition systems that graph comprehension supplies can now be specialized to stuff types, in particular simulations as well as modules.