Girard structures on discrete quantales

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W^*-algebras are supposed to model non-commutative measure theory in the same way that C^*-algebras model non-commutative topology. One might therefore expect that the quantale-theoretic *spectrum* of a W^*-algebra should admit some form of complementation. The most obvious sense in which this could happen is if the spectrum were to admit a cyclic dualizing element, but this only occurs in the finite-dimensional case.

We propose to solve this problem by introducing a concept of *coupled* pair of quantales. We show that every W^*-algebra gives rise to a *Girard couple* (i.e., coupled quantales equipped with a cyclic dualizing element). Another example of a Girard couple arising in quantale theory comes from considering $S \otimes S^*$ and $Hom(S, S)$ for an arbitrary sup-lattice S.

We also show how to construct a single Girard quantale from a Girard couple, and applying this construction to the latter example, we obtain a Girard quantale whose sublattice of right-sided elements is isomorphic to S.

Most of our results appear as applications of theory of *-autonomous categories on sup-lattices.

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