

# Computads and weak higher dimensional categories

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Computads were introduced by Ross Street. They were re-introduced and renamed as polygraphs by Albert Burroni (*Theoretical Computer Science* 115 (1993)). The following two papers reproduce the basic definitions, and are relevant to my subject in essential ways: [Jacques Penon, “Approche polygraphique des inf-catégories non strictes”, *Cahiers TGDC*, vol. XL-1 (1999)]; and [Francois Metayer, “Resolutions by polygraphs”, *TAC* 11, no.7, 2003].

Multitopic categories are a proposal for the still-contested general concept of weak higher dimensional category (WHDC). References are: [C. Hermida, M. Makkai and J. Power *JPAA* 153 (2000), 157 (2001) and 166 (2002)]; and [M. Makkai, “The multitopic omega-category of all multitopic omega-categories”, 1999 and 2004, [www.math.mcgill.ca/makkai](http://www.math.mcgill.ca/makkai)].

The connection between computads and multitopic categories is given by the fact that the category of multitopic sets is equivalent to a full subcategory of the category of computads, namely the one whose objects are the so-called many-to-one computads. This fact is proved in the paper [V. Harnik, M. Makkai and M. Zawadowski, “Multitopic sets are the same as many-to-one computads”, June 2002, [www.math.mcgill.ca/makkai](http://www.math.mcgill.ca/makkai)].

A natural question is whether one could use arbitrary computads in place of the many-to-one ones in a new definition of WHDC's. The answer is “yes”; and in fact, the definition becomes simpler because of the added flexibility gained by removing the “many-to-one” restriction from computads. The new definition uses a notion of equivalence of cells within an omega-category that is very natural but seems not to have been mentioned in the literature; the notion is called coinductive equivalence for a good reason. The interesting thing that happens is that (Theorem) the new notion of WHDC thus obtained turns out to be equivalent to that of ordinary, strict omega category. Thus, the new definition is a bad one, since we know that even 3D WHDC's (tricatagories) are not equivalent to 3-categories.

The proof of the theorem uses a very interesting “path-object” construction in Francois Metayer's above mentioned paper. I take the theorem to be an important justification for the concept of multitopic category. It also seems to be a first one among results that are highly desired (at least by me), ones that rigorously compare proposed notions of WHDC's, and thus show that the game of defining the concept of WHDC is not completely rule-free.