Läuchli's completeness theorem
from a topos-theoretic perspective

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We prove a variant of Läuchli's completeness theorem for intuitionistic predicate calculus [3]. (See also [2], [1], [5] and [4].) The formulation of the result relies on the observation (due to Lawvere) that Läuchli's theorem is related to the logic of the canonical indexing of the atomic topos of $\mathbf{Z}$-sets. We show that the process that transforms Kripke-counter-models into Läuchli-counter-models is (essentially) the inverse image of a geometric morphism. Completeness follows because this geometric morphism is an open surjection.

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