Kan extensions for double categories

Robert Pare *

Two dimensional category theory is category theory based on Cat, the category of categories. One of the insights provided by double category theory is that Cat should be considered as a double category with functors and profunctors as arrows. Thus it is important to understand its completeness properties. It transpires that limits are only lax functorial. Of course the whole story of limits must include Kan extensions, which are parametrized limits. We show that companions and conjoints (a kind of adjointness between horizontal and vertical arrows) are special cases of Kan extensions, and that these together with limits are sufficient for constructing Kan extensions along double functors satisfying a kind of Conduche condition. This is the best that can be expected as the right adjoint for "pulling back" along a functor appears as a special case. Thus we take the existence of such Kan extensions as our notion of completeness in double categories.

*Joint work with Marco Grandis.