Quantale-valued sets, quantale modules, and groupoid actions

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In addition to the two traditional ways of understanding sheaves on a space B, namely as local homeomorphisms $X \to B$ and as presheaves $\Omega(B)^{\mathrm{op}} \to \mathrm{Set}$, there is a third way, according to which sheaves are regarded as locale-valued sets $S \times S \to \Omega(B)$ [1], which has been at least implicit in several generalizations of the notion of sheaf where locales are replaced by more general quantales (or even quantaloids), namely in works by Borceux, van den Bossche, Gylys, Mulvey, Nawaz, van der Plancke, Stubbe, Walters, Zamora Ramos, etc. This talk has two purposes. First, I will describe, for involutive quantales, a fourth elementary way of understanding sheaves, namely as quantale modules equipped with a "Hilbert basis" — this view is interesting even for sheaves on locales because it provides an (at least metaphorical) analogue of Swan's theorem for local homeomorphisms instead of vector bundles. And I will show, using the correspondence between groupoids and quantales of [2], that examples of sheaves on quantales are obtained canonically from sheaves (on locales) equipped with a groupoid action.

REFERENCES

[1] M.P. Fourman and D.S. Scott, Sheaves and logic, In: *Applications of Sheaves* (Proc. Res. Sympos. Appl. Sheaf Theory to Logic, Algebra and Anal., Univ. Durham, Durham, 1977), Lecture Notes in Math., 753 (1979), 302–401.

[2] P. Resende, Étale groupoids and their quantales, *Adv. Math.*, 208 (2007), 147–209.

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