Projections of weak Hopf algebras and weak Yang-Baxter operators

R. González Rodríguez *

In this talk we show that, for a weak Hopf algebra projection $g: B \to H$, the subalgebra of coinvariants B_H of B, defined in [1], is a Hopf algebra in the category ${}^{H}_{H}\mathcal{YD}$, i.e., the category of Yetter-Drinfeld modules defined by Böhm in [4]. Also, we complete this result, obtaining a one to one correspondence between Hopf algebras in the category ${}^{H}_{H}\mathcal{YD}$ and projections of weak Hopf algebras $g: B \to H$ ([2] and [3]).

To prove these results, in [3] we introduce the notions of weak Yang-Baxter operator and weak braided Hopf algebra and we show that it is possible to construct non-trivial examples of this algebraic structures using Hopf algebras in ${}^{H}_{H}\mathcal{YD}$ where the antipode of H is an isomorphism. The definition of weak braided Hopf algebra, generalizes the one introduced by Takeuchi in [8], and as a particular instances we obtain the definition of weak Hopf algebra (see [5]) and, if the weak Yang-Baxter operator is the braiding of a braided category, the new notion of weak Hopf algebra in a braided setting.

Finally, we want to emphasize that these results give a good weak Hopf algebra interpretation of well-known theorems proved by Radford [7], Majid [6] and others in the Hopf algebra setting, that provides a correspondence between Hopf algebra projections and Hopf algebras in the category of Yetter-Drinfeld modules.

References

- J.N. Alonso Álvarez, R. González Rodríguez, Crossed products for weak Hopf algebras with coalgebra splitting, J. of Algebra 281 (2004) 731-752.
- [2] J.N. Alonso Álvarez, R. González Rodríguez, J.M. Fernández Vilaboa, Yetter-Drinfeld modules and projections of weak Hopf algebras, to appear in J. of Algebra (2006).
- [3] Alonso Álvarez, J.N., González Rodríguez, R., Fernández Vilaboa, J.M., Weak Hopf algebras and weak Yang-Baxter operators, preprint (2006).
- [4] G. Böhm, Doi-Hopf modules over weak Hopf algebras, Comm. in Algebra, 28 (2000) 4687-4698.
- [5] G. Böhm, F. Nill, K. Szlachányi, Weak Hopf algebras, I. Integral theory and C^{*}-structure, J. of Algebra, 221 (1999) 385-438.

^{*}Joint work with J.N. Alonso Álvarez and J.M. Fernández Vilaboa.

- [6] S. Majid, Cross products by braided groups and bosonization, J. of Algebra, 163 (1994) 165-190.
- [7] D.E. Radford, The structure of Hopf algebras with a projection, J. of Algebra, 92 (1985) 322-347.
- [8] M. Takeuchi, Survey of braided Hopf algebras, Contemporary Math., 267 (2000) 301-323.