A term calculus for cartesian differential categories

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Last year, at the Calgary CMS meeting and at the White Point CT2006 meeting, we introduced "differential categories" and "cartesian differential categories". Differential categories are a re-development of the notion of differential following the linear-logic-inspired approach due to Thomas Ehrhard, where a comonadic structure is used to account for two types of maps ("smooth" and "linear"), with a combinator D acting on smooth maps to form linear ones in the suitable way. The distinction between diffrerential categories and cartesian differential categories is one of perspective: in the cartesian differential case we work in an "abstract coKleisli category", so both cartesian and monoidal tensor structure is available to us. The intention is to characterize those categories that arise as the coKleisli categories of differential categories, and whose differential structure is induced from the differential structure of the base category.

One complication of working in the abstract coKleisli setting is that the mix of cartesian and monoidal structures can make deciding coherence questions (such as proving diagrams commute) harder. We have a good technology of circuit (or string) diagrams, but they are cleaner in the monoidal context, and become less helpful with cartesian structure. A tool we are finding useful in working with such problems is a term calculus for the maps of these categories — in effect such a calculus allows us to (almost) pretend we are doing first year calculus calculations, and certainly seems to make deciding coherence questions simpler.

In the talk I will briefly review the basic definitions of differential and cartesian differential categories, with some illustrations, and then will outline the term calculus, indicate how one shows it to be complete and sound, and illustrate its use in proving diagrams commute.

This is work in progress, and the exact nature of the material presented will probably be decided at the last minute!

References

[1] R.F. Blute, J.R.B. Cockett, R.A.G. Seely. *Differential Categories*, Mathematical Structures in Computer Science 16(2006) 1049–1083.

*Joint work with R.F. Blute and J.R.B. Cockett.