# Topology for $V$-categories and $(T, V)$-categories

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We treat a commutative unital quantale $V$ as a symmetric monoidal-closed category and first study $V$-enriched categories. Being simultaneously order-enriched, a topological category over Set, and a symmetric monoidal-closed category, the category $V$-Cat has an immensely rich structure that is best exploited when studied in conjunction with the category $V$-Mod of $V$-categories and of $V$-(bi-)modules between them. For example, every $V$-category comes with a strictly categorically-motivated but topologically equally interesting closure operation that leads naturally to notions of separation and completion, as first studied by Bill Lawvere in his famous 1973 Milano paper. We will present a synopsis of the topological aspects of the $V$-Cat/$V$-Mod interplay and then demonstrate which steps are needed to rescue the essential parts of the theory at the more general $(T, V)$-level, where now $T$ is a Set-monad that comes with a lax extension to $V$-Cat.