The equivalence between Barr-Beck cotriple homology and the Brown-Ellis higher Hopf formulae

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We use Categorical Galois Theory to interpret cotriple homology in arbitrary semi-abelian monadic categories in terms of generalized Brown-Ellis-Hopf formulae. Given such a category $\mathcal{A}$ and a chosen Birkhoff subcategory $\mathcal{B}$ of $\mathcal{A}$, thus we describe the Barr-Beck derived functors of the reflector of $\mathcal{A}$ onto $\mathcal{B}$ in terms of centralization of higher extensions. In case $\mathcal{A}$ is the category Gp of all groups and $\mathcal{B}$ is the category Ab of all abelian groups, this yields a new proof for Brown and Ellis's formulae.

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