Calculating limits and colimits compositionally

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The present authors have considered (bi-)categories of spans and cospans of graphs in the study of algebras of processes, cospan operations providing the sequential operations, and span operations corresponding parallel operations (see [1], [2], [3], [4], [5]). This paper is an attempt to answer the question as to why these categories of spans and cospans of graphs arise in applications, with the aim of understanding better the abstract context of this research. First notice that, in our work, graphs appear in different roles, and it is important to distinguish these roles. To this end we consider a category **E** whose objects are "state spaces of systems" – the category of graphs is just one example used very frequently in computer science. We are interested in describing systems constructed from parts. There are two ways of describing how a system is made of parts, one a geometrical description of the configuration of the parts, the other an algebraic description in which the total system is given by an expression in the parts, in an algebra of systems. The geometric point of view commonly consists in describing the system as a (co)limit of subsystems, the (co)limit parametrized by some form of graph (this is the second use of graphs). Our answer to the question raised above is based on [6]: the algebras which allow a system to be described compositionally are the well-supported compact closed (wscc) categories of spans and cospans in **E** ([7]). This result may be regarded as an abstract Kleene theorem.

References

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