

Bicategories and the philosophy of language

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This paper analyses the arguments of a group of philosophers – Davidson [7, 6], Kim [9], Bennett [1] and Parsons [15] – who have worked on the semantics of actions. They argue that inference patterns involving adverbs require a semantics, formalised in first-order logic, in which actions are first-class individuals. In particular, equalities between actions must be meaningful. (We should note, in passing, that this work is semantically focused, and thus it addresses concerns which are rather orthogonal to the syntactic investigations of, for example, Lambek and his co-authors [4, 3, 11, 12, 10].)

The inferences involving adverbs are those of the form

$$\text{Susan ran quickly} \vdash \text{Susan ran}, \quad (1)$$

and we argue that a much less problematic, and more direct, formalisation would be bicategorical. Objects are states, 1-cells are actions (i.e. transitions between states), and 2-cells are inferences such as (1): we have, then, a locally posetal bicategory. In order to handle assertions, we use a 2-fibration [8] in Boolean algebras over this bicategory: objects of the total category are assertions about a state, 1-cells of the total category are sets of physical processes implementing the action, and 2-cells (liftings of inferences such as (1)) are containments between the corresponding sets of physical processes. We also need a 2-comprehension (i.e. a right adjoint to the \top functor from base to total category) in order to construct actions out of sets of physical processes, and we need equality predicates (i.e. right adjoints to diagonals) in order to say when the results of two actions are the same.

We can prove the following. Given the above setup, we can use the results of Carboni and Walters [2] to show that these fibrations arise as subobject fibrations of categories of relations over regular categories. We also give a sequent calculus formulation of inference in the internal language, and prove soundness and completeness.

We can, thus, give a rather direct formalisation of the inferences in question, and thus move the philosophical debate significantly forward. Equalities do turn out to be significant, but they are the equalities which, in the Carboni-Walters construction, come from the right adjoints to diagonals: in philosophical terms, they are equalities between possible worlds, rather than equalities between actions. We have, then, using a methodology very like Lawvere's [13, 14] to analyse the logical infrastructure necessary for inferences such as (1).

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