Kan extensions, models of sketches and the coequalizer of the kernel pair process

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Let $K : \mathbf{B} \to \mathbf{A}$ be a functor such that the image $K(\mathbf{B}_0)$ of the objets in $\mathbf{B}$ is a cogenerating set of objects for $\mathbf{A}$. It will be shown that the coequalizer of the kernel pair process applied to the adjunction $\mathbf{Set}^K \dashv Ran_K : \mathbf{Set}^\mathbf{B} \to \mathbf{Set}^\mathbf{A}$ produces a reflection $\mathbf{Set}^\mathbf{A} \to Mono(\mathbf{Set}^K)$ with stable units and monotone-light factorization. Such is the case if $\mathbf{A}$ is the opposite category of the category of positive ordinals $[n]$, and $\mathbf{B}$ is its full subcategory with just the object $[0]$, giving rise to the (non-trivial) monotone-light factorization for simplicial sets via ordered simplicial complexes.

The result above follows from the more general fact that an adjunction $\mathbf{C} \to \mathbf{X}$ equipped with a (pre)factorization system on $\mathbf{C}$ and satisfying some conditions, produces a reflection with stable units and monotone-light factorization. E.g., the (non-trivial) monotone-light factorization for categories via preorders.

The latter factorization is just the restriction of the former. Why this is so will be studied replacing $\mathbf{A}$ and $\mathbf{B}$ by sketches, i.e., categories with distinguished families of cones.

References

[1] Carboni, A., Janelidze, G., Kelly, G. M., Paré, R., *On localization and stabilization for factorization systems.*, App. Cat. Struct. 5 (1997) 1–58.

[2] Xarez, J. J. *A Galois theory with stable units for simplicial sets*, Theory Appl. Categories 15 (2006) 178–193.