Combinatorial description of many-to-one computads

Marek Zawadowski

The definition of multitopic categories the weak ω-categories in the sense of Makkai contains two ingredients. The first constitutes a description of shapes of cells that are considered (this includes the relation between cells and their domains and codomains) and the second constitutes a mechanism of composition. This work is meant to be a contribution to a better understanding of the first ingredient of the M. Makkai's definition of multitopic category.

In my talk I want to present a combinatorial description of many-to-one computads (see [3]). The main device to do this is the notion of an *ordered face structure*. Ordered face structures are to many-to-one computads are like simple ω-graphs, c.f. [1], to the ono-to-one computads i.e. the free ω-categories over ω-graphs. In other words ordered face structures represent the shapes of arbitrary cells in such many-to-one computads. They are axiomatized using operations of domain δ, codomain γ, and a strict order $<^\sim$. The most important axiom is the axiom of *globularity*

$$\gamma\gamma(a) = \gamma\delta(a) - \delta\dot{\delta}^{-\lambda}(a), \;\; \delta\gamma(a) \equiv_1 \delta\delta(a) - \gamma\dot{\delta}^{-\lambda}(a),$$

which is the many-to-one version of the usual globularity condition $cc = cd$ and $dc = dd$. The category of ordered face structures is a monoidal globular category in the sense of M. Batanin, with tensor given by local pushouts with a suitable extended order relation $<^\sim$. This observation plays the central role in the construction of many-to-one computads.

References

[1] M. Makkai and M. Zawadowski, *Disks and duality*. TAC 8(7), 2001, 114–243.

[2] M. Zawadowski, *On positive face structures and positive-to-one computads*, preprint, 2006, `http://www.mimuw.edu.pl/ zawado/PCI.pdf` .

[3] M. Zawadowski, *On ordered face structures and many-to-one computads*, preprint, 2007, `www.mimuw.edu.pl/ zawado/MCI.pdf` .