2-FILTERED INVERSE LIMITS OF TOPOI

Eduardo J. Dubuc

CT 2007 (Cavoeiro June 2007)

FILTERED

$$\Gamma \xrightarrow{\ F\ } \mathcal{S}et$$

$$\to F_\alpha \longrightarrow F_\beta \longrightarrow \cdots \qquad \mathcal{L}(F)$$

T 1) COLIMIT

$$\begin{array}{ccc} F_\alpha & \xrightarrow{\ \lambda_\alpha\ } & \mathcal{L}(F) \\ & \searrow^{h_\alpha} \quad \equiv & \big\downarrow \exists! \\ & & \mathcal{X} \end{array}$$

I FINITE SET

T 2)

$$\mathcal{L}(F^I) \xrightarrow[\approx]{\text{bijection}} \mathcal{L}(F)^I$$

$$\mathcal{L}(F^I) \xleftarrow{\ (\lambda^I)_\alpha\ } F_\alpha^I \xrightarrow{\ (\lambda_\alpha)^I\ } \mathcal{L}(F)^I$$

$$\overset{\text{FILTERED}}{\mathcal{A}} \xrightarrow{\ F\ } \mathbf{Cat}$$

$$FA \longrightarrow FB \longrightarrow \cdots \qquad \mathcal{L}(F)$$

T 1): COLIMIT

$$\begin{array}{ccc} FA & \xrightarrow{\lambda_A} & \mathcal{L}(F) \\ & \underset{h_A}{\searrow} \ \cong & \downarrow \\ & & \mathcal{X} \end{array}$$

$$\mathbf{Cat}[\mathcal{L}(F), \mathcal{X}] \xrightarrow{\ \cong\ } pCones[F, \mathcal{X}]$$

EQUIVALENCE

T 2): $\mathbb{I}$ FINITE CATEGORY

$$\begin{array}{ccc} \mathcal{L}(F^{\mathbb{I}}) & \xrightarrow{\ \cong\ } & \mathcal{L}(F)^{\mathbb{I}} \\ {\scriptstyle (\lambda^{\mathbb{I}})_A} \nwarrow & & \nearrow {\scriptstyle (\lambda_A)^{\mathbb{I}}} \\ & FA^{\mathbb{I}} & \end{array}$$

$\Downarrow$

T 3): T 1) HOLDS FOR FINITE LIMIT PRESERVING FUNCTORS AND CATEGORIES WITH FINITE LIMITS

RECEPIE FOR CONSTRUCTION OF COLIMITS OF CATEGORIES

Ob $\mathcal{L}(F)$

(x, A) $\quad x \in FA$

Fl $\mathcal{L}(F)$ (PREMORPHISMS)

$$\begin{array}{c} (x, A) \\ \downarrow \xi \\ (y\ B) \end{array} \qquad \begin{array}{ccc} A & & \\ & \searrow^{u} & \\ & & C \\ & \nearrow_{v} & \\ B & & \end{array} \qquad \begin{array}{c} u_* x \\ \downarrow \xi \in FC \\ v_* y \end{array}$$

need COMPOSITION and

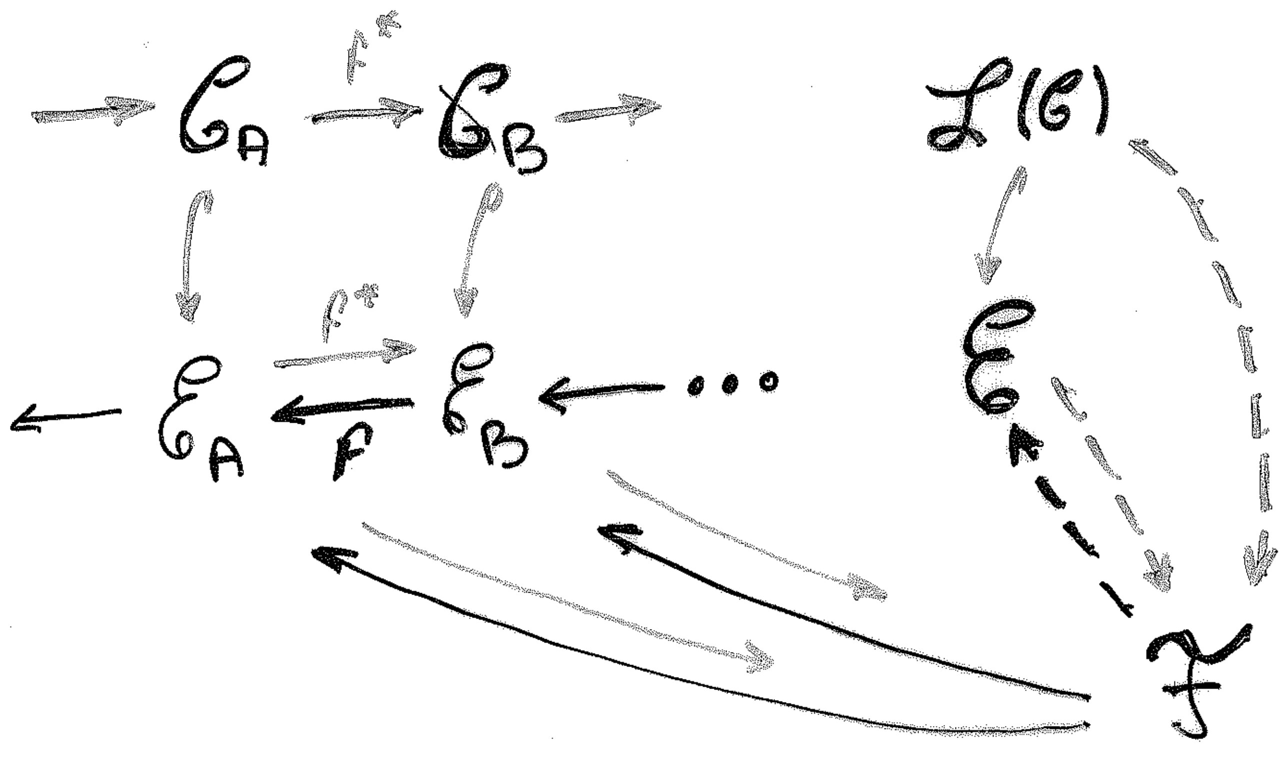
APROPIATE EQUIVALENCE RELATION ON PREMORPHISMS

WORKS IF $\mathcal{A}$ FILTERED

T3) $\Longrightarrow$

SGA4 (SLN Vol 270)

CONSTRUCTION OF FILTERED INVERSE LIMITS OF TOPOI



NEEDS T1 and T2 TO HAVE T3

NOT NECESSARIALY $\mathcal{A}$ FILTERED ?

LOCALLY CONNECTED TOPOS

$$\mathcal{A} \hookrightarrow \mathcal{C} \hookrightarrow \mathcal{E}, \qquad \mathcal{E} \xrightarrow{\gamma} \mathcal{S}$$

$\mathcal{A}$: Galois Objects; $\mathcal{C}$: Connected Objects

GALOIS TOPOS if $\mathcal{A}$ generates

$$\mathcal{C}_A \subset \mathcal{C} \qquad X \mid \gamma^* S \times A \underset{\approx}{\xrightarrow{\sigma}} X \times A$$

$$\mathcal{G}_A \xrightarrow{P_A^*} \mathcal{S} \qquad (X, S, \sigma) \qquad Fl: X \xrightarrow{f} Y \quad S \xrightarrow{\eta} T$$

$$\begin{array}{ccc} \gamma^* S \times A & \underset{\approx}{\xrightarrow{\sigma}} & X \times A \\ \downarrow & & \downarrow f \times id \\ \gamma^* T \times A & \underset{\approx}{\xrightarrow{\sigma}} & Y \times A \end{array}$$

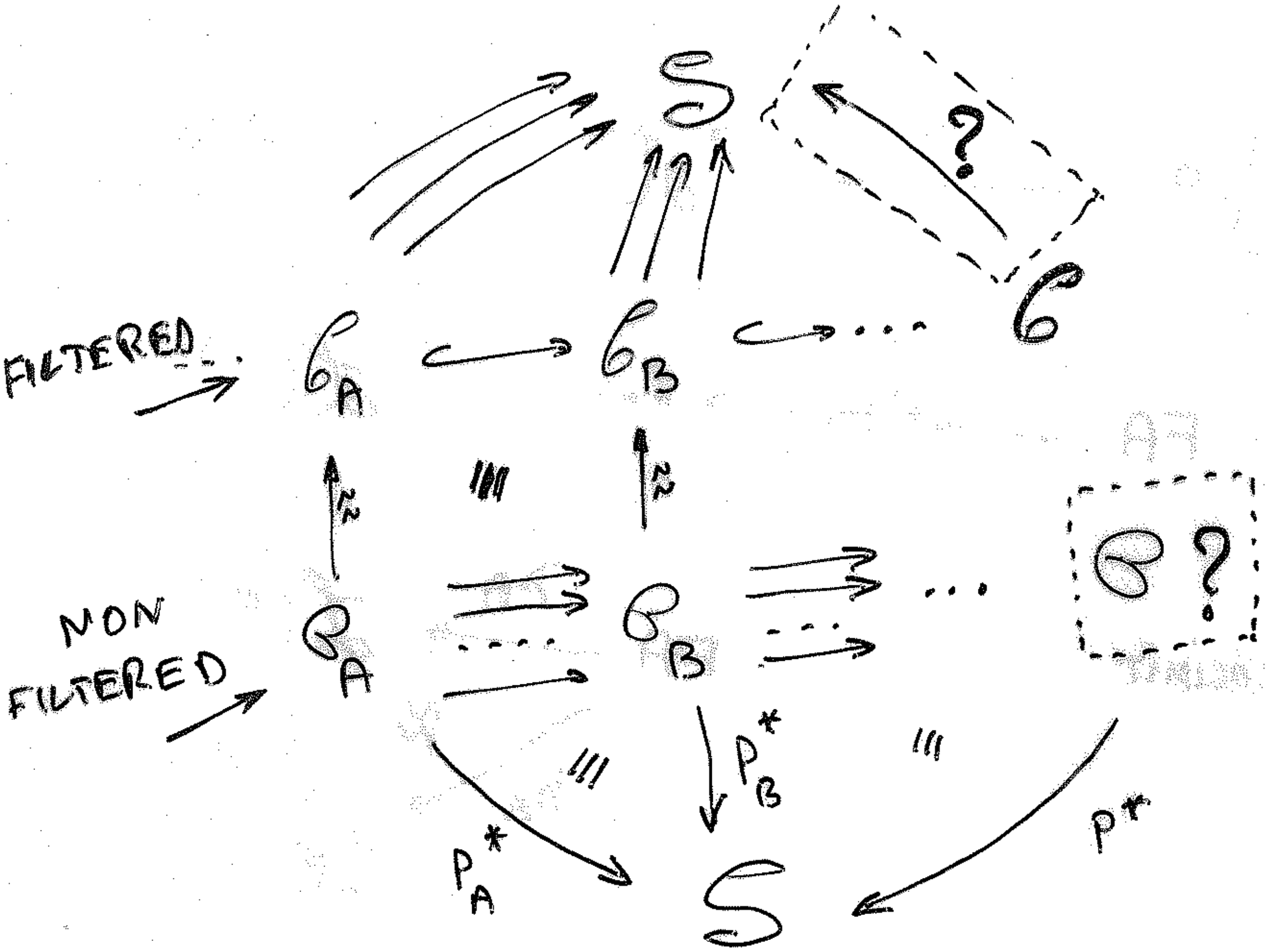
A connected $\Rightarrow$ given f $\exists$ η

$$B \underset{v}{\overset{u}{\rightrightarrows}} A$$

$$\begin{array}{ccc} \mathcal{C}_A & \hookrightarrow & \mathcal{C}_B \\ \uparrow \approx & & \uparrow \approx \\ \mathcal{G}_A & \underset{v^*}{\overset{u^*}{\rightrightarrows}} & \mathcal{G}_B \end{array}$$

YES: $C \dashrightarrow A$, $C \dashrightarrow B$

$C \dashrightarrow B \rightrightarrows A$ NO



$$\mathrm{Points}(\mathcal{C}_A) \longleftarrow \mathrm{Points}(\mathcal{C}_B) \longleftarrow \cdots \quad \mathrm{Points}(\mathcal{C})$$

↳ Filtered inverse limit of sets $\neq \emptyset$?

<u>Problem</u>

CAN NOT CHOSSE A NATURAL FAMILY $\mathcal{C}_A \longrightarrow \mathcal{P}_A$ OF INVERSE EQUIVALENCES

$B \underset{v}{\overset{u}{\rightrightarrows}} A \qquad \mathcal{P}_A \underset{v^*}{\overset{u^*}{\rightrightarrows}} \mathcal{P}_B \;\; (\approx \Downarrow \theta_{vu}) \qquad \exists\, \theta_{vu}$

$\theta_{uu} = id$

$\theta_{wv} \circ \theta_{vu} = \theta_{wu}$

$\theta_{vu}\, \theta_{v'u'} = \theta_{u'u\, v'v}$

$\theta_{vu} \circ \theta_{uv} = id$

Add a symbol θ_{vu}

$A \underset{v}{\overset{u}{\rightrightarrows}} B \;\; (\Downarrow \theta_{vu})$

$\mathcal{A}$ BECOMES a 2-CATEGORY

RIGTH GENERALITY

DEFINITION OF 2-FILTERED 2-CATEGORY

FF1.

A CONSTRUCTION OF 2-FILTERED BICOLIMITS OF CATEGORIES

Edvardo J. DUBUC - ROSS STREET

arXiv: math.CT/0605304 v 1 11 May 2006

Published in CAHIERS

Á la mémoire de Charles Ehresmann

RESULTS :

a) RECEPIE FOR CONSTRUCTION OF COLIMITS WORKS

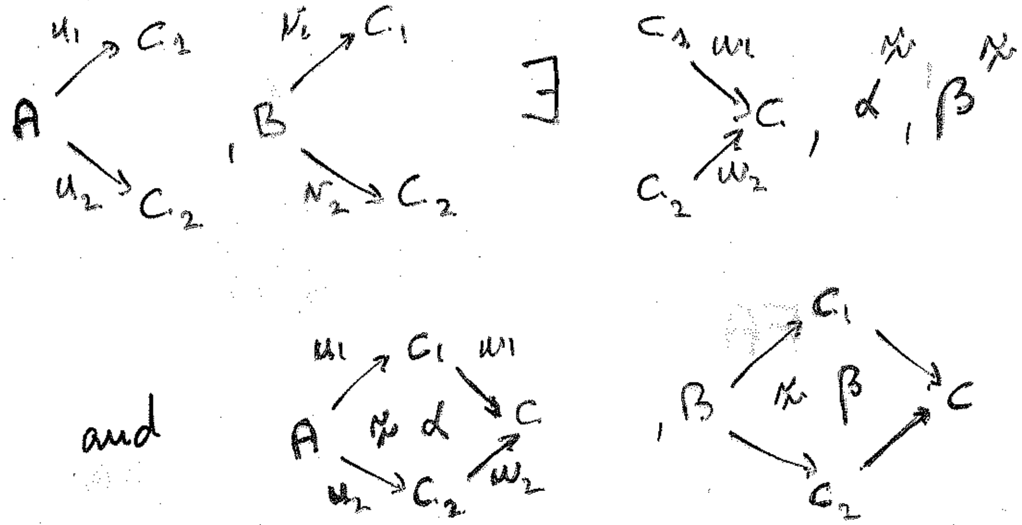
b) Theorem T1 holds

c) Theorem T2 holds

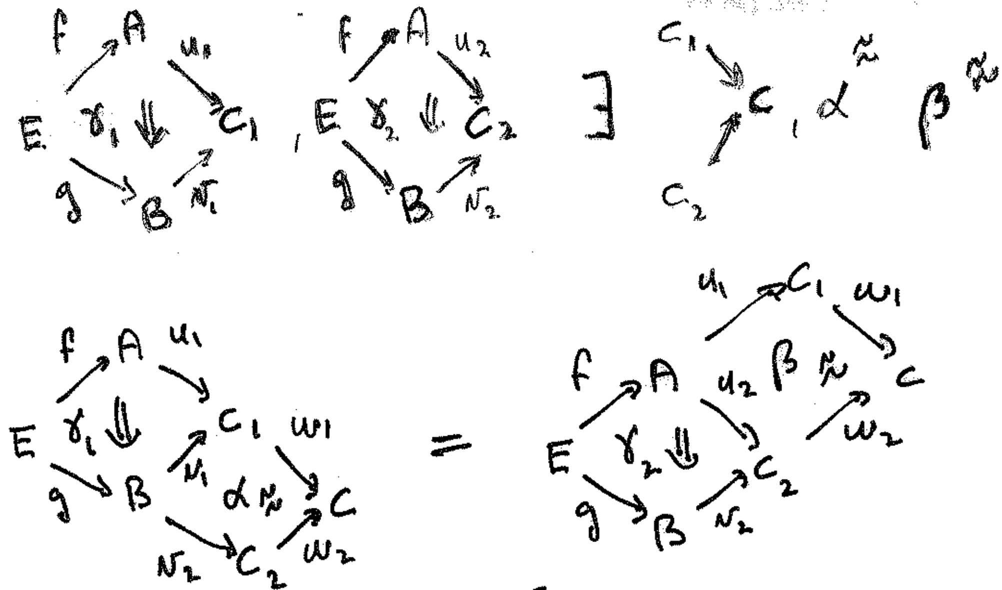
Thus T3 holds

AXIOMS OF 2-FILTERED

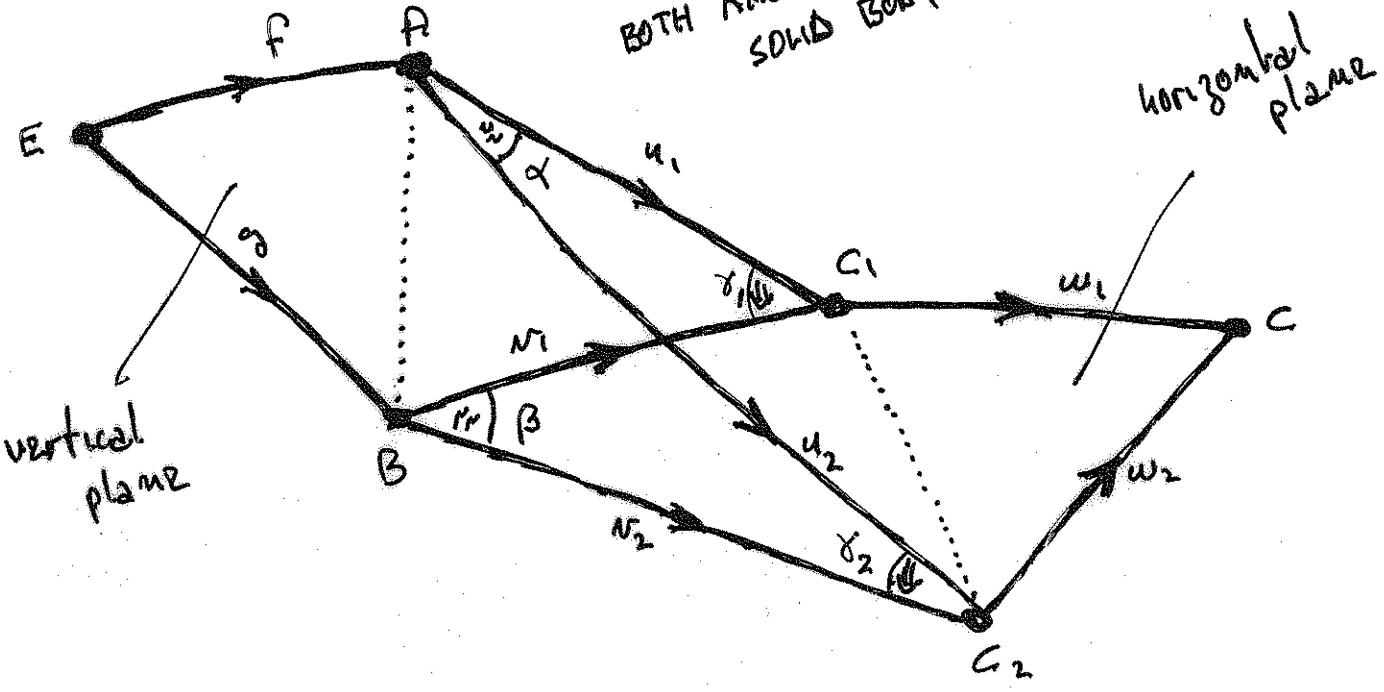
FF1) given



F2) given



BOTH AXIOMS IN ONE SOLID BODY



SGA 4 method for the construction of filtered inverse limits of TOPOI works for non filtered 2-filtered systems.

Object Θ in page 5 exists, $\Theta \approx \mathcal{E}$ and it is a pointed site for the TOPOS $\mathcal{E}$.

CAN define 2-Pro $(\mathfrak{X})$ for any 2-category $\mathfrak{X}$. CAN define cofinal $\mathcal{A} \longrightarrow \mathcal{B}$. etc etc

---

EXAMPLE COV$(\mathcal{E})$. coverings of a TOPOS with refinements are a 2-FILTERED 2-category

$$\begin{array}{lccc} \text{2-ProObject} & \text{COV}(\mathcal{E}) & \xrightarrow{\text{Cech Nerve}} & SS \\ & \downarrow & & \downarrow \\ \text{Pro Object.} & \text{COV}(\mathcal{E}) & \longrightarrow & \text{Ho}(SS) \end{array}$$

ARTIN-MAZUR