The theory of glueing things on

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Outline

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(NB: Talk notes available at http://www.dpmms.cam.ac.uk/∼rhgg2)

Introduction

Two fundamental approaches to higher-dimensional entities in category theory:

- "Topological": possession of sufficiently many good *properties*;
- "Algebraic": specification of a sufficiently well-behaved *structure*.

First school:

- "Easy" to calculate with (and so get things done!);
- "Hard" to pin things down.

Second school:

- "Easy" to pin things down;
- "Hard" to calculate with.

Now:

- A useful organising tool in the "topological" approach is given by Quillen model categories.
- So if we could "algebraise" the notion of Quillen model category…
- In such a way that all the model category structures we find in nature are examples of this more algebraic notion…
- Then we would have a much better understanding of the link between the two approaches.

We *won't* do this today… but we will get some of the way there!

Weak factorisation systems

A *weak factorisation system* (w.f.s.) on a category $\mathscr{C}$ consists of two classes of maps $\mathscr{L}$ and $\mathscr{R}$, closed under retracts, and satisfying:

- **Factorisation**: every $\mathscr{C}$-map can be decomposed as an $\mathscr{L}$-map followed by an $\mathscr{R}$-map; and
- **Weak orthogonality**: $f \pitchfork g$ for every $f \in \mathscr{L}$ and $g \in \mathscr{R}$,

where $f \pitchfork g$ means that for every commutative square

$$\begin{array}{ccc} A & \xrightarrow{h} & C \\ {\scriptstyle f}\downarrow & & \downarrow{\scriptstyle g} \\ B & \xrightarrow[k]{} & D \end{array}$$

there exists a diagonal fill-in $j\colon B \to C$ making both triangles commute.

Intuitively:

- In an orthogonal factorisation system *à la* Freyd-Kelly, factorisations of morphisms are unique up to isomorphism, and diagonal fill-ins are unique;
- In a weak factorisation system, both factorisations and diagonal fill-ins are unique up to "something weaker."

Example

- A functor $f\colon A \to B$ between categories is an **injective equivalence** if it is both injective on objects and an equivalence of categories.
- A functor $g\colon C \to D$ is an **isofibration** if cartesian liftings exist for all isomorphisms in D.
- There is a weak factorisation system on **Cat** with $\mathscr{L} = \{\text{injective equivalences}\}$ and $\mathscr{R} = \{\text{isofibrations}\}$.

Now, the notion of weak factorisation system fails to be algebraic in two ways:

- We do not *choose* an $(\mathscr{L}, \mathscr{R})$ factorisation for each morphism;
- We do not *choose* a diagonal fill-in for each square

$$\begin{array}{ccc} A & \xrightarrow{h} & C \\ {\scriptstyle f}\downarrow & & \downarrow{\scriptstyle g} \\ B & \xrightarrow[k]{} & D \end{array}$$

  with $f \in \mathscr{L}$ and $g \in \mathscr{R}$.

However, we can overcome each of these by modifying the notion of w.f.s.

Functorial realisations

For any category $\mathscr{C}$, we write

- $\mathscr{C}^{\to}$ for the arrow category of $\mathscr{C}$;
- $\mathrm{dom}, \mathrm{cod} \colon \mathscr{C}^{\to} \to \mathscr{C}$ for the domain and codomain functors;
- $\kappa \colon \mathrm{dom} \Rightarrow \mathrm{cod} \colon \mathscr{C}^{\to} \to \mathscr{C}$ for the canonical natural transformation with components $\kappa_f = f \colon \mathrm{dom}(f) \to \mathrm{cod}(f)$.

A *functorial factorisation* (F, λ, ρ) on a category $\mathscr{C}$ is given by a functor $F \colon \mathscr{C}^{\to} \to \mathscr{C}$ and a factorisation of κ as:

$$\kappa \; = \; \mathrm{dom} \xRightarrow{\;\lambda\;} F \xRightarrow{\;\rho\;} \mathrm{cod}.$$

A *functorial realisation* of a w.f.s. $(\mathscr{L}, \mathscr{R})$ on a category $\mathscr{C}$ is given by a functorial factorisation (F, λ, ρ) such that each $\lambda_f \in \mathscr{L}$ and each $\rho_f \in \mathscr{R}$.

Example

The {injective equivalence, isofibration} w.f.s. on **Cat** admits a functorial realisation:

- Given a functor $f \colon A \to B$, we write Ff for the *iso-comma category* $B \downarrow_{\cong} f$:- the full subcategory of the comma category $B \downarrow f$ spanned by the isomorphisms.
- There is a functor $\lambda_f \colon A \to Ff$ sending a to $(\mathrm{id}_{fa} \colon fa \to fa)$;
- There is a functor $\rho_f \colon Ff \to B$ sending $(\theta \colon b \to fa)$ to b.

And this assignment extends to a functorial factorisation (F, λ, ρ) on $\mathscr{C}$, with every λ_f an injective equivalence and every ρ_f an isofibration.

What remains to address is the failure to *choose* diagonal fill-ins between $\mathscr{L}$-maps and $\mathscr{R}$-maps. To do this, let us change perspective:

- Instead of thinking of "being an $\mathscr{L}$-map" as a *property* of a morphism of $\mathscr{C}$, let us think of it as *extra structure* on that morphism:
- Where this extra structure specifies a coherent choice of diagonal fill-in opposite every $\mathscr{R}$-map.
- (And vice versa.)

To understand how we can do this, let us consider for a moment how an *orthogonal* factorisation system works.

Motivation: orthogonal factorisation systems

An *orthogonal factorisation system* on a category $\mathscr{C}$ consists of two classes of maps $\mathscr{E}$ and $\mathscr{M}$, closed under retracts, and satisfying:

- **Factorisation**: every $\mathscr{C}$-map can be decomposed as an $\mathscr{E}$-map followed by an $\mathscr{M}$-map; and
- **Orthogonality**: $f \perp g$ for every $f \in \mathscr{E}$ and $g \in \mathscr{M}$,

where $f \perp g$ means that for every commutative square

$$\begin{array}{ccc} A & \xrightarrow{h} & C \\ {\scriptstyle f}\downarrow & & \downarrow{\scriptstyle g} \\ B & \xrightarrow[k]{} & D \end{array}$$

there exists a *unique* fill-in $j\colon B \to C$ making both triangles commute.

- For an orthogonal factorisation system on $\mathscr{C}$, any choice of $(\mathscr{E}, \mathscr{M})$ factorisation for each map $f\colon X \to Y$:

$$f \quad \longmapsto \quad X \xrightarrow{\lambda_f} Ff \xrightarrow{\rho_f} Y.$$

  extends in a unique way to a functorial realisation (F, λ, ρ) (because of orthogonality).

- Moreover, such a functorial realisation determines the orthogonal factorisation system completely, as follows:

- Any functorial factorisation (F, λ, ρ) on $\mathscr{C}$ determines a pointed endofunctor (R, Λ) on $\mathscr{C}^{\to}$, with

$$R\left(\begin{array}{c} X \\ \big\downarrow{\scriptstyle f} \\ Y \end{array}\right) = \begin{array}{c} Ff \\ \big\downarrow{\scriptstyle \rho_f} \\ Y, \end{array}$$

and $\Lambda_f \colon f \to Rf$ given by

$$\begin{array}{ccc} X & \xrightarrow{\lambda_f} & Ff \\ {\scriptstyle f}\big\downarrow & & \big\downarrow{\scriptstyle \rho_f} \\ Y & \xrightarrow[\mathrm{id}_Y]{} & Y. \end{array}$$

- Dually, any functorial factorisation (F, λ, ρ) on $\mathscr{C}$ determines a copointed endofunctor (L, Φ) on $\mathscr{C}^{\rightarrow}$, with

$$L\left(\begin{array}{c} X \\ \big\downarrow f \\ Y \end{array}\right) = \begin{array}{c} X \\ \big\downarrow \lambda_f \\ Ff. \end{array}$$

and $\Phi_f \colon Lf \to f$ given by

$$\begin{array}{ccc} X & \xrightarrow{\mathrm{id}_X} & X \\ {\scriptstyle \lambda_f}\big\downarrow & & \big\downarrow{\scriptstyle f} \\ Ff & \xrightarrow[\rho_f]{} & Y. \end{array}$$

Moreover, if (F, λ, ρ) is a functorial realisation of an orthogonal factorisation system $(\mathscr{E}, \mathscr{M})$, then:

- The corresponding pointed endofunctor (R, Λ) underlies an idempotent monad $\mathbb{R} = (R, \Lambda, \Pi)$ on $\mathscr{C}^{\to}$;
- The corresponding copointed endofunctor (L, Φ) underlies an idempotent comonad $\mathbb{L} = (L, \Phi, \Sigma)$ on $\mathscr{C}^{\to}$;
- The category of $\mathbb{R}$-algebras is the full subcategory of $\mathscr{C}^{\to}$ spanned by the $\mathscr{M}$-maps;
- The category of $\mathbb{L}$-algebras is the full subcategory of $\mathscr{C}^{\to}$ spanned by the $\mathscr{E}$-maps.

Natural weak factorisation systems

A *natural weak factorisation system* (n.w.f.s.) [Grandis-Tholen 2006] on a category $\mathscr{C}$ is given by:

- A functorial factorisation (F, λ, ρ) on $\mathscr{C}$;
- An extension of the corresponding copointed endofunctor (L, Φ) to a comonad;
- An extension of the corresponding pointed endofunctor (R, Λ) to a monad.

All subject to one additional axiom.

[From the above data, we can define a natural transformation $\Delta \colon LR \Rightarrow RL$: and the extra axiom ensures that Δ gives a distributive law.]

Stated in a more obviously algebraic manner, a n.w.f.s. on a category $\mathscr{C}$ is given by:

- A comonad $\mathbb{L} = (L, \Phi, \Sigma)$ on $\mathscr{C}^{\to}$;
- A monad $\mathbb{R} = (R, \Lambda, \Pi)$ on $\mathscr{C}^{\to}$;
- A distributive law $\Delta\colon LR \Rightarrow RL$.

satisfying some laws:

$$
\begin{aligned}
&\mathrm{dom} \cdot L = \mathrm{dom}, && \mathrm{cod} \cdot L = \mathrm{dom} \cdot R, && \mathrm{cod} \cdot R = \mathrm{cod}; && \\
&\mathrm{dom} \cdot \Phi = 1_{\mathrm{dom}}, && \mathrm{cod} \cdot \Phi = \kappa \cdot R, && \mathrm{dom} \cdot \Lambda = \kappa \cdot L, && \mathrm{cod} \cdot \Lambda = 1_{\mathrm{cod}}; \\
\text{and } &\mathrm{dom} \cdot \Sigma = 1_{\mathrm{dom}}, && \mathrm{cod} \cdot \Sigma = \mathrm{dom} \cdot \Delta, && \mathrm{dom} \cdot \Pi = \mathrm{cod} \cdot \Delta, && \mathrm{cod} \cdot \Pi = 1_{\mathrm{cod}}.
\end{aligned}
$$

This is our "algebraisation" of the notion of weak factorisation system:

- The *property* of being an $\mathscr{L}$-map is replaced with the *structure* of being a coalgebra for the comonad $\mathbb{L}$;
- The *property* of being an $\mathscr{R}$-map is replaced with the *structure* of being an algebra for the monad $\mathbb{R}$;
- The cofree functor $\mathscr{C}^{\to} \to \mathbb{L}$-**Coalg** sends a map of $\mathscr{C}$ to the left half of its $(\mathbb{L}, \mathbb{R})$-factorisation;
- The free functor $\mathscr{C}^{\to} \to \mathbb{R}$-**Alg** sends a map of $\mathscr{C}$ to the right half of its $(\mathbb{L}, \mathbb{R})$-factorisation;
- Liftings between $\mathbb{L}$-coalgebras and $\mathbb{R}$-algebras are built canonically by interacting their (co)algebraic structure.

Example

Consider again the functorial factorisation

$$f \;=\; A \xrightarrow{\lambda_f} B \downarrow_{\cong} f \xrightarrow{\rho_f} B$$

on **Cat**. This extends to a n.w.f.s. $(\mathbb{L}, \mathbb{R}, \Delta)$, for which:

- Giving an $\mathbb{L}$-coalgebra structure on $f \colon A \to B$ amounts to giving an adjoint pseudo-inverse functor $p \colon B \to A$ for which the counit isomorphism $\varepsilon \colon pf \cong \mathrm{id}_A$ is actually an identity;
- Giving an $\mathbb{R}$-algebra structure on $g \colon C \to D$ amounts to making it into a split isofibration.

Building weak factorisation systems

In a locally presentable category $\mathscr{C}$, any set J of maps generates a weak factorisation system $(\mathscr{L}, \mathscr{R})$ where:

$$\mathscr{R} = \{\, g \colon C \to D \mid f \pitchfork g \text{ for all } f \in J \,\}$$
$$\mathscr{L} = \{\, f \colon A \to B \mid f \pitchfork g \text{ for all } g \in \mathscr{R} \,\}.$$

Intuitively:

- Each map $f\colon A \to B$ in J specifies a valid "boundary" shape (A) together with a "cell" (B) which fills it: the map f being the boundary inclusion.
- Each $\mathscr{L}$-map in the resultant weak factorisation system is built by recursively glueing in "J-cells" along their "boundaries" (and then taking retracts).

Building natural weak factorisation systems

[G. 2007] For a locally presentable category $\mathscr{C}$, any set J of maps in $\mathscr{C}$ generates a natural weak factorisation system $(\mathbb{L}, \mathbb{R}, \Delta)$ where:

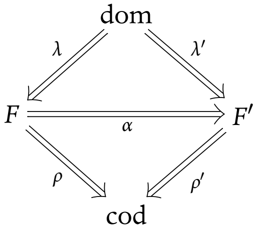
- $\mathbb{R}$-algebras are maps $g\colon C \to D$ of $\mathscr{C}$ equipped with *chosen* liftings against every element of J.
- $\mathbb{L}$-algebras are maps $f\colon A \to B$ of $\mathscr{C}$ equipped with a specification of *how* we recursively glued in "J-cells" along their "boundaries" (think of computads).

We say that $(\mathbb{L}, \mathbb{R}, \Delta)$ is *cofibrantly generated* by J.

(NB: can weaken the local presentability requirement to deal with categories like **Top** and **Haus**.)

Cofibrantly generated n.w.f.s.'s: abstractly

Let $(\mathbb{L}, \mathbb{R}, \Delta)$, $(\mathbb{L}', \mathbb{R}', \Delta')$ be n.w.f.s.'s on $\mathscr{C}$. A *morphism of n.w.f.s.'s* between them is a natural transformation α between the underlying functorial factorisations:



which is compatible with the comonad structures of $\mathbb{L}$ and $\mathbb{L}'$ and the monad structures of $\mathbb{R}$ and $\mathbb{R}'$.

We thus obtain a category $\mathbf{Nwfs}(\mathscr{C})$ of n.w.f.s.'s on $\mathscr{C}$; and can now define a "semantics" functor

$$\mathscr{G}\colon \mathbf{Nwfs}(\mathscr{C}) \to \mathbf{CAT}/\mathscr{C}^{\to}$$

sending a n.w.f.s. $(\mathbb{L}, \mathbb{R}, \Delta)$ to the object

$$(\mathbb{L}\text{-}\mathbf{Coalg} \xrightarrow{U_{\mathbb{L}}} \mathscr{C}^{\to})$$

of $\mathbf{CAT}/\mathscr{C}^{\to}$.

- Given a set J of arrows of $\mathscr{C}$, we may view J as a discrete subcategory of $\mathscr{C}^{\to}$;
- Thus we obtain an object $(J \hookrightarrow \mathscr{C}^{\to})$ of $\mathbf{CAT}/\mathscr{C}^{\to}$;
- And the n.w.f.s. cofibrantly generated by J is now obtained as a reflection of $(J \hookrightarrow \mathscr{C}^{\to})$ along the functor $\mathscr{G} \colon \mathbf{Nwfs}(\mathscr{C}) \to \mathbf{CAT}/\mathscr{C}^{\to}$.

(So we have a n.w.f.s. $(\mathbb{L}, \mathbb{R}, \Delta)$ on $\mathscr{C}$, and a map

$$\begin{array}{ccc} J & \xrightarrow{\ \eta\ } & \mathbb{L}\text{-}\mathbf{Coalg} \\ & \searrow \quad \swarrow_{U_{\mathbb{L}}} & \\ & \mathscr{C}^{\to} & \end{array}$$

of $\mathbf{CAT}/\mathscr{C}^{\to}$, which is universal amongst all such arrows.)

Cofibrantly generated n.w.f.s.'s: concretely

Suppose we are given a generating set J of maps in our (locally presentable) category $\mathscr{C}$, and a map $g\colon C \to D$ of $\mathscr{C}$ which we wish to factorise:

- We consider the set S_g of all squares of the form

$$\begin{array}{ccc} A & \xrightarrow{h} & C \\ {\scriptstyle f}\downarrow & & \downarrow{\scriptstyle g} \\ B & \xrightarrow[k]{} & D \end{array}$$

  where f is in our generating set J.

- We can view each such element $x \in S_g$ as a map

$$(h_x, k_x)\colon\ f_x \to g$$

of $\mathscr{C}^{\to}$; so taking their copairing, we obtain a square

$$\begin{array}{ccc}
\sum_{x \in S_g} A_x & \xrightarrow{\langle h_x \rangle_{x \in S_g}} & C \\
{\scriptstyle \sum_x f_x} \downarrow & & \downarrow {\scriptstyle g} \\
\sum_{x \in S_g} B_x & \xrightarrow[\langle k_x \rangle_{x \in S_g}]{} & D.
\end{array}$$

- Now we factorise this square as

$$\begin{array}{ccccc}
\sum_{x\in S_g} A_x & \xrightarrow{\langle h_x\rangle_{x\in S_g}} & C & \xrightarrow{\mathrm{id}_C} & C \\
{\scriptstyle f_x}\downarrow & & \downarrow{\scriptstyle \lambda^1_g} & & \downarrow{\scriptstyle g} \\
\sum_{x\in S_g} B_x & \longrightarrow & \ulcorner F^1 g & \xrightarrow[\rho^1_g]{} & D.
\end{array}$$

  where the left square is a pushout.
- We obtain in this way the component at g of a functorial factorisation (F^1, λ^1, ρ^1) on $\mathscr{C}$.

Now let (R^1, Λ^1) be the pointed endofunctor on $\mathscr{C}^{\to}$ corresponding to (F^1, λ^1, ρ^1). We have that:

- An algebra for (R^1, Λ^1) is precisely a morphism of $\mathscr{C}$ equipped with a *chosen* lifting against every element of J;
- The algebraically-free monad on the pointed endofunctor (R^1, Λ^1), if it exists, will have the same algebras as (R^1, Λ^1);
- And hence will give us the correct "monad" half of our n.w.f.s.

Moreover, the assumption of local presentability (or whatever) ensures that the algebraically-free monad $\mathbb{R}$ on (R^1, Λ^1) exists (cf. [Kelly, 1980]).

This explains how we obtain the "monad" half of our n.w.f.s.: so what about the "comonad" half?

Let (L^1, Φ^1) be the copointed endofunctor on $\mathscr{C}^{\to}$ corresponding to (F^1, λ^1, ρ^1). Then:

- (L^1, Φ^1) already underlies a comonad on $\mathscr{C}^{\to}$ – it is a *density* (or *model-induced*) comonad in the 2-category $\mathbf{CAT}/\mathscr{C}$.
- The "free monad" construction we used above can be refined to carry the comonad structure of (L^1, Φ^1) along with it.
- (Rather than forming a free monoid in a monoidal category of endofunctors, we form a free monoid in a monoidal category of comonads).

In explicit terms, $\mathbb{R}$ sends $g\colon C \to D$ to the colimit of the following transfinite sequence:

$$\begin{array}{ccccccc}
C & \xrightarrow{\lambda_g^1} & F^1 g & \xrightarrow{\lambda_g^2} & F^2 g & \longrightarrow & \cdots \\
{\scriptstyle g}\downarrow & & \downarrow{\scriptstyle \rho_g^1} & & \downarrow{\scriptstyle \rho_g^2} & & \\
D & \xrightarrow[\mathrm{id}_D]{} & D & \xrightarrow[\mathrm{id}_D]{} & D & \longrightarrow & \cdots
\end{array}$$

where

- $F^2 g$ is given by the following coequaliser:

$$F^1 g \underset{F^1(\lambda_g^1, \mathrm{id}_D)}{\overset{\lambda^1_{\rho_g^1}}{\rightrightarrows}} F^1(\rho_g^1) \longrightarrow F^2 g$$

  and λ_g^2 is the common composite from left to right;
- Each subsequent $F^{\alpha^+} g$ appears as a quotient of $F^1 F^\alpha g$;
- For γ a limit ordinal, $F^\gamma g$ is the colimit of the previous stages.

Compare this with the "small object argument" used to construct factorisations in cofibrantly generated w.f.s.'s.

We begin in the same manner, constructing the functorial factorisation $(F^1, \lambda^1, \rho^1)\dots$

But now, for a map $g \colon C \to D$, we form the following transfinite sequence:

$$
\begin{array}{ccccccc}
C & \xrightarrow{\lambda^1_g} & F^1 g & \xrightarrow{\lambda^1_{\rho^1_g}} & F^1 F^1 g & \xrightarrow{\lambda^1_{\rho^1_{\rho^1_g}}} & \cdots \\
{\scriptstyle g}\downarrow & & \downarrow{\scriptstyle \rho^1_g} & & \downarrow{\scriptstyle \rho^1_{\rho^1_g}} & & \\
D & \xrightarrow[\mathrm{id}_D]{} & D & \xrightarrow[\mathrm{id}_D]{} & D & \longrightarrow & \cdots
\end{array}
$$

And this sequence *almost never* converges! Instead, we are forced to choose a point (sufficiently far along the sequence) at which we will stop.

Why is this interesting?

Let $\mathscr{C} = \mathbf{SSet}$ and let J be the set of horn inclusions.

- J cofibrantly generates a n.w.f.s. $(\mathbb{L}, \mathbb{R}, \Delta)$; and the monad $\mathbb{R}$ restricts to a "fibrant replacement monad" $\mathbb{T}$ on $\mathscr{C}$.
- $\mathbf{AlgKan} := \mathbb{T}\text{-}\mathbf{Alg}$ has:
  - **Objects**: Kan complexes with *chosen* fillers for every horn;
  - **Morphisms**: maps of **SSet** commuting strictly with the chosen liftings.
- So we have a category of algebraic Kan complexes and *strict* maps: which has nice categorical properties (e.g., complete and cocomplete).

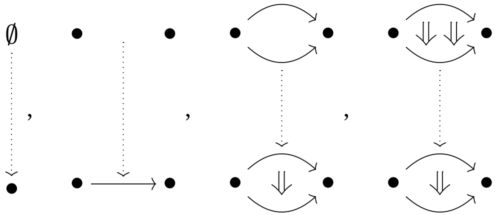
- We get a "pseudo map" category to accompany this "strict map" category by factorising the forgetful functor $U\colon \mathbf{AlgKan} \to \mathbf{SSet}$ as

$$\mathbf{AlgKan} \xrightarrow{J} \mathbf{AlgKan}_\psi \xrightarrow{V} \mathbf{SSet}$$

where J is bijective on objects and V is fully faithful.

- In fact, $\mathbf{AlgKan}_\psi$ is just the co-Kleisli category for the comonad on $\mathbb{T}$-$\mathbf{Alg}$ induced by the free/forgetful adjunction with $\mathbf{SSet}$;
- Hence J has a left adjoint $(\ )'\colon \mathbf{AlgKan}_\psi \to \mathbf{AlgKan}$.

Now let $\mathscr{C} = \mathbf{2\text{-}Cat}$ and let J be the following set of maps:



(cf. [Lack, 2002]).

- J cofibrantly generates a n.w.f.s. $(\mathbb{L}, \mathbb{R}, \Delta)$; and the comonad $\mathbb{L}$ restricts to a "cofibrant replacement comonad" $\mathbb{Q}$ on **2-Cat**.

- This comonad $\mathbb{Q}$ classifies pseudomorphisms: for 2-categories A and B, there is a bijection between

$$\text{pseudo-functors } A \to B \quad \text{and} \quad \text{2-functors } QA \to B.$$

- Hence the co-Kleisli category of $\mathbb{Q}$ is the 2-category $\mathbf{2\text{-}Cat}_{\psi}$ of 2-categories and pseudo-functors.
- We can extend this example in an obvious way to 3-, 4-, . . . , ω-categories, and so obtain a workable definition of weak higher dimensional functor, by *defining*

$$\text{weak functors } A \to B \quad \text{to be} \quad \text{strict functors } QA \to B.$$

Even better, the comonad structure on Q ensures that these weak functors have a strictly associative composition!

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