CO-REPRESENTATIONS AND HOMOLOGY WITH COEFFICIENTS OF LEIBNIZ n-ALGEBRAS

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Leibniz n-algebras were introduced in [3] as a non-skew-symmetric version of Nambu algebras, which naturally arose in the so called Nambu mechanics [5]. They are $\mathbb{K}$-vector spaces $\mathcal{L}$ equipped with a n-linear bracket $[-,\dots,-]:\mathcal{L}^{\otimes n}\to\mathcal{L}$ satisfying the following fundamental identity

$$[[x_1,x_2,\dots,x_n],y_1,y_2,\dots,y_{n-1}]=\sum_{i=1}^{n}[x_1,\dots,x_{i-1},[x_i,y_1,y_2,\dots,y_{n-1}],x_{i+1},\dots,x_n] \qquad (1)$$

In case $n=2$ the identity (1) is the Leibniz identity, so a Leibniz 2-algebra is a Leibniz algebra. In addition, if the bracket is skew symmetric, we recover the definition of Lie algebra. Moreover, the category ${}_n\mathbf{Leib}$ of Leibniz n-algebras is related with the category $\mathbf{Leib}$ of Leibniz algebras by means of the Daletskii's functor [4] $\mathcal{D}_{n-1}:{}_n\mathbf{Leib}\to\mathbf{Leib}$ which assigns to a Leibniz n-algebra $\mathcal{L}$ the Leibniz algebra $\mathcal{L}^{\otimes n-1}$ with bracket given by

$$[a_1\otimes\dots\otimes a_{n-1},b_1\otimes\dots\otimes b_{n-1}]=\sum_{i=1}^{n}a_1\otimes\dots\otimes[a_i,b_1\otimes\dots\otimes b_{n-1}]\otimes\dots\otimes a_{n-1}$$

In the present talk we introduce the notion of co-representation for Leibniz n-algebras, which is equivalent to the category of left modules over the universal enveloping algebra introduced in [2]. This notion in case $n=2$ gives the corresponding notion of co-representation for Leibniz algebras in [6]. Then we construct a complex of a Leibniz n-algebra $\mathcal{L}$ over a co-representation M by means of the Leibniz complex of $\mathcal{L}^{\otimes n-1}$ over the co-representation $\mathrm{M}\otimes\mathcal{L}$. In case $n=2$ we obtain the Leibniz homology developed in [6]. When M is the trivial co-representation $\mathbb{K}$, then we obtain the homology with trivial coefficients [1].

We prove the vanishing of the homology over free objects and a result which generalizes the following isomorphism $HL_\star(\mathcal{L},\mathcal{L})\cong HL_{\star+1}(\mathcal{L},\mathbb{K})$ for Leibniz algebras to Leibniz n-algebras.

References

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