Graph-theoretic Fibring of Logics

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Abstract

It is well known that interleaving presentations is at the heart of fibring, as shown by the mechanism of fibring languages and deduction systems. This idea is abstractly introduced herein at the level of the general notion of m-graph (that is, a graph where each edge can have a finite sequence of nodes as source). Signatures, interpretation structures and deduction systems are seen as m-graphs. After defining a category freely generated by a m-graph, formulas and expressions in general can be seen as morphisms. Moreover, derivations involving rule instantiation are also morphisms. Soundness and completeness results are proved. As a consequence of the generality of the approach our results apply to very different logics encompassing, among others, substructural logics as well as logics with nondeterministic semantics and subsume all logics endowed with an algebraic semantics. Graph-theoretic fibring allows the explicit construction of the interpretation structure resulting from the fibring of a pair of interpretation structures. Soundness and weak completeness are proved to be preserved under very general conditions. Strong completeness is also shown to be preserved under tighter conditions. In this setting, the collapsing problem appearing in several combinations of logic systems is avoided.

For more details see the preprints

A. Sernadas, C. Sernadas, J. Rasga, and M. Coniglio. Graph-theoretic fibring of logics Part I - Completeness. Preprint, SQIG - IT and IST - TU Lisbon, 1049-001 Lisboa, Portugal, 2008. Submitted for publication.

A. Sernadas, C. Sernadas, J. Rasga, and M. Coniglio. Graph-theoretic fibring of logics Part II - Completeness preservation. Preprint, SQIG - IT and IST - TU Lisbon, 1049-001 Lisboa, Portugal, 2008. Submitted for publication.

`http://wslc.math.ist.utl.pt/s84.www/cs/clc/search_bib.html`