Categorical logics extending Birkhoff's equational logic

Lurdes Sousa

In a category of algebras the satisfaction of an equation, or an implication, is nothing else than injectivity with respect to a convenient regular epimorphism (as observed in [5]). Departing from this fact (and from ideas of Roşu [7]) we formulated injectivity and orthogonality logics with respect to morphisms of a category in two versions: a finitary one and a general one ([4, 1, 2, 3]).

In reasonable categories the finitary orthogonality and injectivity logics are sound and complete, and they encompass the equational logic of Birkhoff and the implicational logic of Quackenbush [6].

Concerning the general injectivity and orthogonality logics, which are always sound, we obtained several completeness theorems.

I am going to present these logics and their relationship with the Orthogonal Subcategory Problem and the Small Object Argument.

If time permits I will talk about ongoing work on a different approach of the categorical finitary logic.

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