

# Faithfulness and the coequalizer of the kernel pair process

João J. Xarez

Abstract.

Starting from a ground structure consisting of an adjunction  $\mathbf{C} \rightarrow \mathbf{X}$  and a prefactorization system  $(\mathcal{E}, \mathcal{M})$  which factorizes the unit morphisms  $\varphi_C : C \rightarrow GF(C)$ ,  $C \in \mathbf{C}$ , through an epimorphism  $\eta_C$ , a full epireflection  $\mathbf{C} \rightarrow \mathbf{M}$  is obtained with unit  $\eta : 1_{\mathbf{C}} \rightarrow HI$ . A reflective factorization system is associated, and there may be a concordant-dissonant (in a similar sense to the one referred in [3]) and also a monotone-light factorization. We will show that in the case of any adjunction  $\mathbf{Set}^{\mathbf{A}} \rightarrow \mathbf{Set}^{\mathbf{B}}$ , given by right Kan extensions along a functor  $K : \mathbf{B} \rightarrow \mathbf{A}$ , there is a monotone-light factorization which coincides with the concordant-dissonant one, provided the objects in the image of the functor  $K$  are a cogenerating set for  $\mathbf{A}$ . Remark that this condition is equivalent to demanding that the composition  $\mathbf{Set}^K \cdot y$  with the Yoneda embedding is a faithful functor. A generalization of the given results, to left adjoints from presheaves into a cocomplete category, is then possible.

## REFERENCES

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