

Internal actions via cross-effects of functors: general theory and conjugation
action on proper subobjects
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Abstract

This is joint work with Manfred Hartl. We develop a notion of cross-effects on functors in a general categorical context, which generalizes the original concept which was introduced by Eilenberg and MacLane in the case of functors between abelian categories, and was adapted to functors with values in the category of groups by Baues and Pirashvili. The consideration of the cross effects on the identity functor on a finitely cocomplete category \mathbb{C} allows us to define a notion of an internal action of an object G on an object A of \mathbb{C} , which is functorially equivalent with a point in \mathbb{C} over G , hence to Bourn and Janelidze’s characterization of actions as algebras over a certain monad on \mathbb{C} in the semi-abelian case. We also use cross effects to define higher categorical commutators. We show that any proper subobject of an object A in \mathbb{C} admits a “conjugation” action of A , and that this characterizes proper subobjects if \mathbb{C} is semi-abelian, a result which has also been proved independently by Mantovani and Metere. We more generally show that if \mathbb{C} is semi-abelian, then for subobjects X, Y of some object A , X is proper in the supremum of X and Y if and only if X is stable under the restriction to Y of the conjugation of A on itself. This amounts to a “direct” proof of Bourn and Janelidze’s result, i.e. a proof which does not use Beck’s criterion. We also show that the two axioms of algebras which characterize actions in this context may be replaced by axioms expressed in terms of cross-effects.

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