On categories with semidirect products

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Abstract. The categorical notion of semidirect product was introduced by Bourn and Janelidze in [1]. A category $\mathbf{C}$ with split pullbacks is said to be a category with semidirect products if, for every morphism $p: E \to B$ in $\mathbf{C}$, the pullback functor $p^*: \mathsf{Pt}(B) \to \mathsf{Pt}(E)$ has a left adjoint and is monadic.

In this note we consider the case where the category $\mathbf{C}$ is pointed, has coequalizers of reflexive pairs and binary coproducts. Forming the category of internal actions (as in [2]) we have by definition that $\mathbf{C}$ has semidirect products if the category $\mathsf{Pt}(C)$ of points is equivalent to the category $\mathsf{Act}(C)$ of internal actions.

It is well known that: (a) a variety of universal algebras has semidirect products if and only if it is protomodular; (b) every semiabelian category has semidirect products; (c) not every homological category has semidirect products.

This talk is divided in two parts. In the first part we analyze the monadicity of p^* and give some necessary and sufficient conditions for a category $\mathbf{C}$ to have semidirect products. In the second part we introduce the notion of strict action and show that $\mathbf{C}$ has semidirect products if and only if it is protomodular and every internal action is strict.

References

[1] D. Bourn and G. Janelidze, *Protomodularity, descent, and semidirect products*, Theory Appl. Categ. **4** (1998), no. 2, 37–46.

[2] G. Janelidze, *Internal crossed modules*, Georgian Math. J. **10** (2003), no. 1, 99–114.