Wellfounded coalgebras

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For an endofunctor H an initial algebra I has, by Lambek's Lemma, invertible structure map, thus, it can be considered as a coalgebra. We call an H-coalgebra *wellfounded* if it has a coalgebra homomorphism into I. This is equivalent to the wellfoundedness in Taylor's "Practical Foundations", defined by means of an induction principle. And it is also equivalent to the recursivity studied by Capretta, Uustalu and Vene: a coalgebra is recursive if it has unique coalgebra-to-algebra homomorphism into every H-algebra. Finally, we also prove that the dual concept of completely iterative algebra is equivalent to wellfoundedness. All these results hold for all endofunctors H preserving preimages.

*Joint work with Stefan Milius.

Initial normal covers in bi-implicative toposes

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The existence of semi-direct products in a semi-abelian category induces a notion of action of an object G on an object X. This determines a corresponding contravariant functor

$$\mathsf{Act}_X\colon \mathcal{C} \to \mathsf{Set},\ G \mapsto \{\xi | \xi \text{ is a } G\text{-action on } X\}.$$

In various cases of interest, this functor is representable, for example by the group of automorphisms of X (for $\mathcal{C}$ the category of groups) or the Lie algebra of derivations of X (for $\mathcal{C}$ the category of Lie algebras), and so on.

When $\mathcal{E}$ is a topos, the dual of the corresponding category $\mathcal{E}_*$ of pointed objects is both semi-abelian and arithmetical. We prove first that the representability of actions on an object X in this semi-abelian category reduces to the existence of an initial normal cover of X in $\mathcal{E}_*$, that is, the existence of an initial object in the category of normal epimorphisms with codomain X.

When the object X is decidable, the problem is somehow trivial since the initial normal cover of X is simply the identity on X. This is in particular the case in every boolean topos. And in the case of a Grothendieck topos, we prove that a necessary (resp. sufficient) condition for the existence of initial normal covers is the fact that an infinite pullback of normal epimorphisms is still an epimorphism (resp. normal epimorphism). When the topos is bi-implicative – that is, the lattices of subobjects are both Heyting and co-Heyting algebras – the necessary condition is also sufficient. This is in particular the case in every topos of presheaves.

*Joint work with Dominique Bourn.

Glueing monads: a descent problem

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Motivated by a desire to gain a better understanding of the "dimension-by-dimension" decompositions of certain prominent monads in higher category theory, we investigate descent theory for endofunctors and monads. After setting up a basic framework of indexed monoidal categories, we describe a suitable subcategory of **Cat** over which we can view the assignment $\mathbf{C} \mapsto Mnd(\mathbf{C})$ as an indexed category; on this base category, there is a natural topology. Then we single out a class of monads which are well-behaved with respect to reindexing. The main result is now, that such monads form a stack. Using this, we can shed some light on the free strict ω-category monad on globular sets and the free operad-with-contraction monad on the category of collections.

Commutators and Galois theory

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It has already been shown by various authors that the commutator introduced by Smith characterizes particular categorical central extensions (namely, those extensions that are central with respect to the subcategory of abelian objects). We investigate the converse question, whether categorical Galois theory can be used to define a notion of commutator, which does not only imply the classical examples, but also, for example, the Peiffer commutator of crossed modules.

Homology of precrossed modules and Galois theory

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The adjunction between crossed modules and precrossed modules over a fixed group B can be seen as a special case of a more general adjunction between internal groupoids and internal reflexive graphs in a Maltsev variety. By using the categorical Galois theory and the theory of commutators we characterize the central extensions [2] with respect to this latter adjunction, getting in particular a description of the central extensions of precrossed modules and of precrossed rings. Then the classical Stallings-Stammbach five term exact sequence associated with a short exact sequence of groups is shown to exist in a general categorical context. As an application, we extend a result on the lower central series of precrossed modules due to Conduché and Ellis [1].

References

[1] D. Conduché and G. J. Ellis, *Quelques propriétés homologiques des modules précroisés*, J. Algebra 123, 1989, 327-335.

[2] G. Janelidze and G. M. Kelly, *Galois theory and a general notion of central extension*, J. Pure Appl. Algebra 97, 1994, 135-161.

*Joint work with Tomas Everaert.

Descent on 2-fibrations and strongly 2-regular 2-categories

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We consider pseudo-descent in the context of 2-fibrations. A 2-category of descent data is associated to a 3-truncated simplicial object in the base 2-category. A morphism q in the base induces (via *comma-objects* and pullbacks) an internal category whose truncated simplicial nerve induces in turn the 2-category of descent data for q. When the 2-fibration admits direct images, we provide the analogous of the Beck-Bénabou-Roubaud theorem, identifying the 2-category of descent data with that of pseudo-algebras for the pseudo-monad $q^*\Sigma_q$. We introduce a notion of *strong 2-regularity* for a 2-category $\mathcal{R}$, so that its basic 2-fibration of internal fibrations $cod : \mathsf{Fib}(\mathcal{R}) \to \mathcal{R}$ admits direct images. In this context, we show that *essentially-surjective-on-objects* morphisms, defined by a certain lax colimit, are of effective descent by means of a Beck-style pseudo-monadicity theorem.

Quillen model category structures for homotopy of internal categories

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The aim of this talk is to describe Quillen model category structures on the category of internal categories and functors in a given finitely complete category.

Under suitable mild restrictions we obtain a model category structure which turns out to resemble very much Strøm's model category structure on the category of topological spaces and continuous maps.

We are also interested in the case in which the ambient category is semi-abelian, since then these model category structures induce a notion of homotopy of internal crossed modules. In the case of the category of groups and homomorphisms, we obtain model category structures on the category of crossed modules of groups.

*Joint work with T. Everaert and T. Van der Linden.

The Thompson groups

Tom Leinster [*]

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In the 1960s Richard Thompson discovered three groups, F, T and V, with several remarkable properties. F in particular turned out to be one of those mathematical objects that arises in all sorts of unexpected places; it is somehow implicit in the concept of associativity. But for mysterious reasons the group theorists have concealed the fact that F and V have extremely simple categorical descriptions. I will expose their secret.

[*]Joint work with Marcelo Fiore.

Homology and term rewriting systems

Philippe Malbos

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In this talk I introduce a homological finiteness condition for the finite convergence of one-sorted and first-order equational theories. The construction of this condition is connected to a simulation of term rewriting systems by rewriting systems on algebraic theories in the sense of Lawvere. The homology of one of these is given in term of Baues-Wirsching homology with coefficients in cocartesian natural systems. Using Fox calculus, a complete term rewriting system gives rise to a free acyclic resolution of a cartesian natural system. I will prove that any equational theory admitting a specification by a finite complete term rewriting system satisfy the homological condition FP_3.

Minimization of Quantum Automata

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A new notion of quantum automaton, which generalizes that of Chris Moore, is given. The automata states are density operators on a Hilbert space and its outputs are probability spaces over the spectrum of an observable. A suitable notion of quantum behaviour is introduced and the problem of finding its minimal realization is addressed using techniques from linear automata theory.

*Joint work with A. M. Martins and A. Sernadas.

Elgot Algebras.
A Base for Denotational Semantics

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Denotational Semantics is usually based on algebras with additional structure (order, metric, etc.) which makes it possible to interpret recursive specifications in the algebra. Calvin Elgot and his group have proposed to use completely iterative theories, i. e., theories in which certain recursive specifications are requested to have unique solutions, as the basis for denotation semantics. In this talk we propose so-called complete Elgot algebras, which are much easier to work with. A complete Elgot algebra is an algebra of an endofunctor H with a choice of a solution for every flat system of recursive equations. This choice is required to satisfy two simple and well-motivated axioms: functoriality (stating that the given choice of solutions is uniform) and CIA identity (stating that simultaneous recursion can be performed sequentially). In comparison with other models in the literature these axioms are simpler. And they stem canonically from completely iterative theories: Elgot algebras are precisely the Eilenberg-Moore algebras of the free completely iterative monad on H. It is well-known that this monad assigns to each object X the terminal coalgebra of the functor $H(-) + X$. Examples of Elgot algebras include continuous algebras on cpos, algebras on complete metric spaces with contracting operations, and all completely iterative algebras (cia), i. e., algebras in which every system of flat recursive equations has a unique solution.

Localic germ groupoids of inverse semigroups

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Groupoids and inverse semigroups, which cater for more general notions of symmetry than groups, have many applications in algebra and geometry, and they are related in many ways — in particular, there are several constructions of topological groupoids from inverse semigroups. Based on the correspondence, which I shall recall, between localic étale groupoids and quantales that has been established in [1], in this talk I shall study the groupoid of germs of a pseudo-group, showing that its construction can be extended to any inverse semigroup whose idempotents form a frame, yielding a localic groupoid whose spectrum is, in the case of, say, the pseudo-group of partial homeomorphisms of a Hausdorff space, the usual topological germ groupoid. The construction we provide can be carried over to an arbitrary topos (immediately yielding, for instance, a G-equivariant construction via an interpretation in the topos of G-sets), and the germ groupoid obtained is universal in the sense that, as a quantale, it is the image of an inverse semigroup by a left adjoint functor.

References

[1] P. Resende, Étale groupoids and their quantales, arXiv:math/0412478.

Galois Theory and Double Central Extensions

Valentina Rossi

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We define a Galois structure Γ between central extensions and extensions in any Mal'cev variety [3]. By using two commutator conditions, we introduce the notion of double central extension that extends the one given by Janelidze in the case of the variety of groups [2]. Thanks to the theory of commutators we then prove that the covering morphisms relative to this Galois structure are precisely the double central extensions [1]. The equivalence of Γ-coverings and double central extensions gives to this last notion its full meaning in any Mal'cev variety.

References

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[2] G. Janelidze, *What is a double central extension? (The question was asked by Ronald Brown)*, Cah. Topologie Géom. Différ. Catég. 32, No 3, 1991, 191-201.

[3] J.D.H. Smith, *Mal'cev Varieties,* Lect. Notes Math. 554, Springer-Verlag, 1976.

Every topological category is a category of lax algebras

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Recently, a number of topological categories have been shown to be categories of lax algebras. Hence these lax algebras could provide a convenient framework for universal topology. To decide how suitable they are for this task, the question

"Which categories arise as categories of lax algebras?"

should be settled.

We propose scenes as an axiomatic framework in which to study lax algebras. Then we show that every fibering in the sense of Manes, and hence in particular every topological category, arises as a category of lax algebras for a suitable scene.

Weak factorization systems

Walter Tholen *

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Weak factorization systems are a standard tool in abstract homotopy theory, as part of the notion of Quillen model structure. In this talk we shall attempt to give a natural extension of the notion of (functorial) weak factorization system to double categories and illustrate it by examples.

*Joint work with Marco Grandis (Genova), in progress.

Homotopy vs. homology of simplicial objects

Tim Van der Linden

Vrije Universiteit Brussel, Belgium

We investigate in which sense the Mal'tsev axiom implies that the simplicial homotopy relation is an equivalence relation, and relate this to pointed and unpointed notions of homology of simplicial objects.

A convenient setting for obtaining Galois theories with stable units

João J. Xarez

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We will present a very simple construction which produces reflections analogous to the one of categories into preordered sets. For instance, one obtains in this way full reflections having the stable units property either for simplicial groups as for crossed modules (= categories in groups).