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Functoriality and topos representations for quantales of coverable groupoids

In [3] it has been seen that for a well behaved open localic groupoid G (a *coverable* groupoid) there is a strong form of embedding of the quantale $\mathcal{O}$ of G into the quantale Q of an étale groupoid $\hat{G}$ that covers G in the sense that there is a surjective morphism $J : \hat{G} \to G$ which restricts to an isomorphism $\hat{G}_0 \cong G_0$. For instance, any locally compact Hausdorff groupoid in the sense of harmonic analysis [2], regarded as a localic groupoid, is of this kind, and so, in particular, Lie groupoids are coverable. Let us refer to such a pair $(Q, \mathcal{O})$ as a *quantal pair.* The main motivation in [3] has been to provide a quantale-theoretic description of (at least some) open groupoids which, similarly to the situation with étale groupoids, does not require the multiplicativity axiom.

The purpose of this talk is to give an overview of new results that improve our understanding of coverable groupoids and quantal pairs. One set of results concerns the functoriality of the quantal pair associated to a coverable groupoid: an appropriate notion of action for quantal pairs yields an equivalence of categories G-*Loc* $\cong (Q, \mathcal{O})$-*Loc*, where $(Q, \mathcal{O})$ is the quantal pair associated to G, and based on this we obtain quantale-theoretic descriptions of equivariant sheaves on groupoids, principal bundles, Hilsum–Skandalis maps and Morita equivalence in a way that extends the functoriality results for quantales of étale groupoids developed in [6, 5, 7].

Another set of results concerns global element representations of groupoid quantales. For an étale groupoid G the domain map $d : G_1 \to G_0$ equipped with the left G-action given by multiplication is regarded as an object $\mathbf{G}$ of the classifying topos BG, and the quantale Q of G is isomorphic to the quantale of global sections of the internal quantale of binary relations $P(\mathbf{G} \times \mathbf{G})$. This has been previously mentioned in [4] and a written proof appeared in the work of Simon Henry [1]. A reasonable generalization of this for general open groupoids is unlikely to exist, but for a coverable groupoid G, if we now write $\mathbf{G}$ for the domain map $d : G_1 \to G_0$ regarded as an internal locale in BG, the internal sup-lattice tensor product $\mathbf{G} \otimes \mathbf{G}$ yields an internal quantale in BG whose quantale of global elements is isomorphic to the quantale of G.

References:

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*Joint work with Pedro Resende.

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