

The Middle-Interchange Law for Dikranjan-Giuli closure operators

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The categorical notion of closure operator as given by Dikranjan and Giuli in their 1987 paper (*Topology Appl.* 27, 129-143) has been central to a significant body of research work in Categorical Topology, the themes of which range from the problem of characterizing epimorphisms in particular categories of topological spaces and settling the question of cowellpoweredness, to the provision of a general categorical framework which allows for a proof of significant topological results in a rather general context, such as the Tychonoff Theorem.

There has been some work on the structure of the (very large) lattice of the totality of all closure operators on a given category with a fixed subobject structure. Our starting point for this talk is the fact that, as has been known early on, there are two ways of composing closure operators. These two compositions are linked by a lax middle-interchange law, the strict validity of which in special contexts is our subject. In fact, we will explore this question more generally for so-called *factoroids*, which not only include closure operators and orthogonal factorization systems, but also functorial weak factorization structures, as used in Quillen model categories for homotopy theory.