Monadicity and connected components

João J. Xarez

Let $\mathcal{C}$ be a category of algebras for a monad $T = GF$ in **Sets**. It will be shown that, for any full reflection $H \vdash I : \mathcal{C} \to \mathcal{M}$, such that all its unit morphisms are regular epimorphisms and $GHIF(1) \cong 1$, where 1 stands for one point set, the notions of *semi-left-exactness* (*admissibility* in the sense of categorical Galois theory), *stable units* and *left-exactness* (see [1]) are simplified. In such a setting the reflection is admissible if and only if it is *simple* (see [1]), which reduces to *attainability* (in a similar sense to the one given in [3] in the particular case of semigroups). For instance, the reflection of compact Hausdorff spaces into Stone spaces is admissible because connected components of compact Hausdorff spaces are connected; furthermore, it has stable units because any finite product of connected components is connected; but its left adjoint does not preserve all pullbacks since a pullback of connected components is not always connected. These results had their origin in generalizing the proof of Theorem 3 in [2], where it is shown that the reflection of semigroups into semilattices has stable units.

References

[1] Carboni, A., Janelidze, G., Kelly, G. M., Paré, R., *On localization and stabilization for factorization systems*, App. Cat. Struct. 5 (1997) 1–58.

[2] Janelidze, G., Laan, V., Márki, L., *Limit preservation properties of the greatest semilattice image functor*, Internat. J. Algebra Comput. 5 (2008) 853–867.

[3] Tamura, T., *Attainability of systems of identities on semigroups*, J. Algebra 3 (1966) 261–276.