CLP-compactness

or

When "compactness" includes connectedness

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A topological space X is *CLP-compact* if every cover of X consisting of clopen sets has a finite subcover. This generalization of compactness, intensively studied by Šostak and Steprāns, includes in a natural way *connectedness*. We discuss CLP-compactness for topological spaces and topological groups.

A topological group G is CLP-compact iff the quotient $G/q(G)$ is CLP-compact, where $q(G)$ is the quasi-component of G. In this way the study of CLP-compact groups can be reduced to the case of totally disconnected groups. An immediate application of Ellis' theorem shows that CLP-compact totally disconnected groups are maximally almost periodic. This imposes the class of totally bounded groups as a natural enviroenement where CLP-compactness should be studied. Here a usefull criterion for CLP-compactness can be provided that permits to constract many CLP-compact totally disconnected groups that may have an arbitrary dimension given in advance.

The CLP-compactness criterion becomes very transparent and handy in the case of pseudocompact groups. As an application we prove that the product of pseudocompact CLP-compact groups is CLP-compact (the preservation of CLP-compactness under products is one of the main problems in this field).