

# Injectivity in a Category: Well Behaviour Theorems

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To Bernhard, Walter, and Mehdi

On their 80th, 60th, and 60th birthday, respectively

Much of the classical set based universal algebra can be and, if we may say so, should be studied in a general category as well as in the category **Set** of sets. Authors, a few of them have been mentioned in the references, have studied the general notion of injectivity long ago, and some investigated injectivity with respect to special monomorphisms in special categories.

Here we try to refresh the problem in the collection of two papers, referring the known results mainly to Bernhard Banaschewski, who classifies the behaviour of injectivity, and Walter Tholen, who has further studied this notion in a much more categorical setting, and also M. Mehdi Ebrahimi, who describes the relationship between the class  $mod\Sigma$  of models of a set  $\Sigma$  of equations in the category **Set** and the corresponding class  $mod(\Sigma; \mathcal{E})$  of model of  $\Sigma$  in a more general category  $\mathcal{E}$ , with respect to injectivity and some related notions.

In this paper, the three “Well Behaviour Theorems of Injectivity”, so called and studied by the above mentioned authors, are discussed in details. We then bring some examples on the category of acts over semigroups to support these results.

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\*Joint work with M. Haddadi, M. Mahmoudi

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