Injectivity in a Category:
Smallness Conditions

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Some of the so called smallness conditions in Algebra as well as in Category Theory, important and interesting for their own and also tightly related to Injectivity, are Essential Boundedness, Cogenerating Set, and Residual Smallness.

In this paper of the collection of two, first we discuss these smallness conditions and some other notions related to injectivity with respect to an arbitrary subclass $\mathcal{M}$ of morphisms in an arbitrary category $\mathcal{A}$, mainly given by Bernhard Banaschewski and Walter Tholen.

Then, we study these notions, as well as the Well Behaviour of Injectivity, in the class $mod(\Sigma, \mathcal{E})$ of models of a set Σ of equations in a suitable category $\mathcal{E}$, given by M.Mehdi Ebrahimi. The basic nature of these results is that whatever a property holds in $mod\Sigma$ of models of Σ in the category **Set**, the corresponding property holds in $mod(\Sigma, \mathcal{E})$, provided $\mathcal{E}$ satisfies some special properties, in particular when $\mathcal{E}$ is a Grothendieck topos.

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