Cocomplete $\mathcal{T}$-categories, injectivity, and Kan-extensions

Dirk Hofmann

In [2] we introduced the notion of a *topological theory* $\mathcal{T} = (\mathbb{T}, \mathsf{V}, \xi)$ – consisting of a monad $\mathbb{T} = (T, e, m)$, a quantale $\mathsf{V} = (\mathsf{V}, \otimes, k)$ and a map $\xi : T\mathsf{V} \to \mathsf{V}$ – as a possible "syntax" for Topology. In fact, this concept

1. permits us to view several objects of topology (including topological spaces, of course) as generalised enriched categories ($\mathcal{T}$-categories), and
2. provides us with enough structure to carry notions and results of enriched Category Theory into the realm of topology.

In this talk we consider in particular weighted colimits, cocompleteness and adjoint functors; and formulate appropriate counterparts for topological structures. Surprisingly or not, these notions are not disconnected from classical Topology but rather provide us with new perspectives and new arguments. For instance, we show that

1. cocomplete topological spaces (and more generally: cocomplete $\mathcal{T}$-categories) are precisely the injective spaces,
2. the Yoneda map (see [1]) embeds each (topological) space universally into an injective space,
3. the subcategory of injective (=cocomplete) objects and left adjoints is monadic over the category of (separated) objects.

References

[1] Maria Manuel Clementino and Dirk Hofmann, *Lawvere completeness in Topology*, Technical report, University of Aveiro, 2007; arXiv:math.CT/0704.3976.

[2] Dirk Hofmann, *Topological theories and closed objects*, Adv. Math. **215** (2007), 789–824.